HELP TO ARITHMETIC.

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HELP TO ARITHMETIC

DESIGNED FOR THE USE OF SCHOOLS.

BY

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PREFACE.

THE writer of the following pages believes that an apology is due for the apparent want of completeness and consecutiveness that are evident in his treatment of the subject they deal with. His only excuse is that this very inconsecutiveness was intentional. His desire was not to make a book. That would seem unnecessary in the presence of the many very excellent treatises on Arithmetic now current in schools. The present work is intended as a companion to any text-book that may be in use, and the object of the writer has been twofold. He has proposed to himself to exhibit in a compendious form explanations of certain Arithmetical difficulties, which, in the course of many years' teaching, he has found satisfactory to his pupils; and to put before boys examples of sums in various rules fully worked out in a clear, uniform, and natural manner. In the "money rules" he has endeavoured

to show that there are no new processes; only new technical expressions to be explained; and that these rules can therefore be made at once to depend on the ordinary forms of proportion.

It will be seen from the above remarks that the design of the book is of the humblest nature. There are no novelties, no ambitious attempts. It only aspires to be useful, and endeavours to accomplish this end by means of clear explanations, and by urging upon learners the necessity of a neat, logical, and methodical treatment of sums.

It is the intention of the writer to follow up this publication with two or more similar treatises on Algebra, Euclid, and other kindred school subjects of an elementary nature; the whole will thus form a single work, of which the present must be considered the First Part.

CONTENTS.

		PAGE
I.	INTRODUCTION	1
	Numeration and Notation	2
	Division, Definition of	2
	Arithmetical Tables	3
	Square and Cubic Measure	5
	Least Common Multiple	9
II.	Vulgar Fractions	11
III.	Decimal Fractions	28
IV.	Proportion	40
V.	RULES REDUCIBLE TO PROPORTION.	
	Simple Interest	43
	Compound Interest	47
	Discount	50
	Stocks	51
	Profit and Loss	56
VI.	MISCELLANEOUS EXAMPLES.	
	Proportional Parts, &c.	59
	Chain Rule	62
	Per centage	63
	Concluding Remarks	63
NOTES		68

PART I.

ARITHMETIC.

1. IN order to avoid using a new symbol and word for every different number, all nations, who have attained any proficiency in counting at all, have adopted a radix, or turning point, after which no new symbols are used; and farther numbers are expressed by the combination of old symbols, and by the positions which they relatively occupy. This radix has in all civilized nations been the number ten; not that that is the most convenient number, (twelve would be the most convenient), but because the system was adopted while they were still in a savage state, and did not reason on its convenience or inconvenience for elaborate computation. They were led to employ the number ten by reason of the ten fingers of the hand, and no doubt at first counted with small round stones or any similar substances. The words *digit* and *calculation* refer us back to a rude origin of Arithmetic, or rather, of counting. A clear and interesting explanation of numeration and notation will be found in the early part of De Morgan's Arithmetic.

2. *Numeration and Notation.*

Begin at the unit figure and separate the figures into groups of six each. Then the figures in the second group will give the number of millions, in the third group the number of billions, in the fourth group the number of trillions, and so on.

Thus, take the number

12345678905698.

Group it thus:

12,345678,905698.

It will be read as follows:

12 billions, 345 thousand 678 millions, 905 thousand, 698.

N.B. The rule given above is not universally adopted. Some writers, (all continental nations we believe), consider a billion to be a thousand millions, a trillion, a thousand billions, and so on. In this way, the above number would be read, 12 trillions, 345 billions, 678 millions, 905 thousand, 698.

3. *Division.*

The definition of division sometimes given in Arithmetical treatises, is, that it is "the process of finding how often one number is contained in another." This scarcely seems to be sufficient, for it does not include all classes of Arithmetical cases. For abstract numbers it is true, but not necessarily for concrete numbers. In order to take in all cases, and to retain strictly Arithmetical language and ideas, it would seem that two definitions are necessary, as thus.

By the division of one concrete quantity by another *of the same kind* is meant the process of finding how

often the second is contained in the first, or how many parcels, each containing the second number of things, there will be in the first number of things: e.g. 12 apples $\div$ 3 apples. Ans. 4.

By the division of a concrete quantity by an abstract number, is meant the process by which we find, if we separate the first into as many equal parcels as are represented by the second, how many units each parcel will contain: e.g. £2. 16*s*. $\div$ 3. Ans. 18*s*. 8*d*.

We shall return to these definitions in Note B, on the Division of Fractions.

4. *Arithmetical Tables.*

The following tabular results are very important, and are far too often forgotten.

In Avoirdupois weight;

112 lbs. make 1 cwt.
The lb. Av. contains 7000 grs. Tr.

In Length measure;

6 ft. make 1 fathom.

$5\frac{1}{2}$ yds. make 1 rod, pole, or perch.

220 yds. make 1 furlong.
1760 yds. make 1 mile.

In Cloth measure;

4 qrs. make 1 yd.
5 qrs. make 1 English ell.

In measure of Surface;

$30\frac{1}{4}$ sq. yds. make 1 sq. rod, pole, or perch.

4840 sq. yds. make 1 acre.

In Cubic measure;

1728 cubic in. make 1 cubic ft.

The following weights and measures, though not required for ordinary Arithmetic, should be observed.

In Length measure;

12 lines make 1 inch.
9 inches make 1 span.
18 inches make 1 cubit.

$69\frac{1}{9}$ miles make 1 geographical degree.

$69\frac{1}{9}$ miles is the length of $\frac{1}{360}$ part of the circumference of the earth at the equator.

In Cloth measure;

A Flemish ell contains 3 qrs.
A French ell contains 6 qrs.

In Capacity;

A firkin	(of beer)	contains 9 gals.
A kilderkin	"	contains 18 gals.
A barrel	"	contains 36 gals.
A hogshead	"	contains 54 gals.
A pipe	(of wine)	contains 126 gals.

Value of Coins;

A groat = 4*d.*
A tester = 6*d.*
A noble = 6*s.* 8*d.* $\left(= £\frac{1}{3}\right)$.
An angel = 10*s.*
A mark = 13*s.* 4*d.* $\left(= £\frac{2}{3}\right)$.
A moidore = 27*s.*

In Surveying;

Gunter's chain is 22 yds. long, and contains 100 links.

Hence an acre contains 10 square chains.

A cub. foot of distilled water weighs 1000 oz.

A lb. of gold contains 24 carats.

From this fact we express the fineness of gold by saying that it is so many carats fine; i.e. so many parts out of 24. Thus, if a mass of gold were 15 carats fine, $\frac{15}{24}$ of the mass would be gold, and the rest, viz. $\frac{9}{24}$, would be "alloy," a substance depending on the nature of the mass.

Standard gold is 22 carats fine, the remaining two parts being a mixture of silver and copper. Nearly $46\frac{3}{4}$ sovereigns are coined out of a lb. Tr. of standard gold. Jewellers' gold is 18 carats fine.

Standard silver contains 37 parts out of 40 pure silver, the remaining three parts being copper. A lb. Tr. of standard silver yields 66 shillings.

24 pence are coined out of a lb. Av. of copper.

5. *Square and Cubic Measure.*

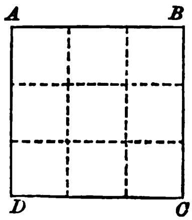
A common or linear yard is length, and measures length.

A sq. yard is surface, and measures surface, or area. It is a square surface whose length and breadth are both a yard. The length round a sq. yard is 4 yards.

A cubic yard is a solid, and measures solidity, or cubic contents. It is a solid whose length, breadth, and height are each a yard.

A cubic yard has 6 surfaces, each of which is a sq. yard.

From these facts it is clear that the ideas of a common, a square, and a cubic yard, are perfectly different. No amount of common yards will make a square yard. There is a very intimate relation between common and square yards, as there is between apples and pence, but as no amount of pence will make apples, so no amount of common yards will make square yards.



When the length table is known the surface table can be formed from it by multiplying each of the figures of the length table by itself.

This is easy to prove. For let $ABCD$ be a square yard. Then both AB and BC are common yards. Therefore each of them consists of three feet. If through the extremities of these feet we draw straight lines parallel to the sides of the square, it is clear that we shall get 9 compartments each of which is a foot long and a foot broad, i.e. we shall have 9 sq. ft.; which proves the statement.

Thus 12 in. make 1 ft.

And 12×12 sq. in. make 1 sq. ft.

Again, $5\frac{1}{2}$ yds. make 1 p.

And $5\frac{1}{2} \times 5\frac{1}{2}$ sq. yds. $\left(\text{i.e } \frac{11}{2} \times \frac{11}{2} = \frac{121}{4} = 30\frac{1}{4}\right)$ make one sq. pole.

So in cubic measure, we must multiply any figure of the common table three times by itself to get the corresponding figure of the cubic table.

For along all the lines of the above figure build walls, and, when the walls are a foot high, lay down a floor; then there will be 9 boxes each a foot long, a foot broad and a foot high; that is, there will be 9 cubic feet. Repeat this till our walls are three feet, or a yard high. There then will be three times 9 or 27 cubic feet in the solid. But the solid is a yard long, a yard broad, and a yard high, that is, it is a cubic yard. Hence there are 27 cubic feet in a cubic yard; which proves the statement.

Thus 12 in. make 1 ft.

And $12 \times 12 \times 12$ cubic in. make 1 cubic ft.

It is easy to prove in exactly the same manner as above, that if we want to find the number of sq. ft. in any area, we must multiply the number of feet in the length by the number of feet in the breadth. So we can find the number of cubic feet in a solid by multiplying the number of feet in the length, the number of feet in the breadth, and the number of feet in the height, together.

It must be remembered that length cannot be multiplied by breadth. We are however at liberty to use the expression "length $\times$ breadth $=$ area," if we understand that we are using an abbreviation, in order to avoid the continual repetition of a long phrase. The same is clearly true for cubic measure.

Length cannot be multiplied by breadth, because that is contrary to the definition of multiplication. Nor could length and breadth be said in any arithmetical sense to make area. We shall return to this point in Note A, on the multiplication of fractions.

The following three formulæ are sufficient for all questions in square and cubic measure:—

$$l \times b = a \quad \text{..........} \quad (1),$$

$$l \times b \times h = c \quad \text{..........} \quad (2),$$

$$a \times h = c \quad \text{..........} \quad (3),$$

where either l, b, h, stand for length, breadth or width, height or depth, or thickness; a stands for area or surface, and c for cubic contents, or solidity.

Ex. There is a room which is 12 feet long, 9 feet broad and 8 feet high. Find

1. The area of the ceiling.
2. The area of the walls.
3. The area of the whole of the interior of the room.
4. The length of the sum of all the edges of the room.
5. The length of a beading round the ceiling.
6. The area of a cornice, three inches broad, in the ceiling round the room.
7. The area of a wainscoating, nine inches broad, in the walls round the floor.
8. The thickness of sand laid evenly over the floor, the cubic dimensions of the sand originally being 8 ft. long, 6 ft. broad and 8 in. deep.
9. The cubic contents of the room.

6. *Least Common Multiple.*

One of two methods must be carefully insisted on. Either the "divisors" must all be *prime*, or care must be taken to divide each number by the largest number that will divide both it and the "divisor" in question. In the latter case the "divisor" must divide one at least of the numbers exactly, in order to *ensure* a correct result.

Ex. L.C.M. of 2, 3, 6, 12, 18, 20, 25, 27, 32, 45.

First Method.

$$\begin{array}{r|l}
2 & \cancel{2},\ \cancel{3},\ \cancel{6},\ 12,\ 18,\ 20,\ 25,\ 27,\ 32,\ 45. \\ \hline
2 & 6,\ \cancel{9},\ 10,\ 25,\ 27,\ 16,\ 45. \\ \hline
3 & \cancel{3},\ \cancel{5},\ 25,\ 27,\ 8,\ 45. \\ \hline
3 & 25,\ 9,\ 8,\ 15. \\ \hline
 & 25,\ 3,\ 8,\ \cancel{5}.
\end{array}$$

Ans. $= 25 \times 3 \times 8 \times 3 \times 3 \times 2 \times 2 = 21600$.

Second method.

$$\begin{array}{r|l}
12 & \cancel{2},\ \cancel{3},\ \cancel{6},\ 12,\ 18,\ 20,\ 25,\ 27,\ 32,\ 45. \\ \hline
3 & 1,\ \cancel{3},\ \cancel{5},\ 25,\ 9,\ 8,\ 15. \\ \hline
 & 25,\ 3,\ 8,\ \cancel{5}.
\end{array}$$

Ans. $= 25 \times 3 \times 8 \times 3 \times 12 = 21600$.

Thus in the 2nd line of the 2nd method, under the 18 we put a 3, because the largest number that will divide both 18 and the "divisor" 12, is 6, and 6 will divide 18 with quotient 3. Again the largest number that will divide both 20 and the "divisor" 12 is 4, and 4 will divide 20 with quotient 5: so that we place a 5 under the 20. And so on.

It is clear that the second method is shorter, but requires

considerably more thought and care. The first method is perfectly safe.

There is a third method, which may be called the method of inspection, which is far the quickest and simplest, but perhaps it is better not to teach it *as a rule* till the elements of Algebra have been mastered. It consists in writing down the continual product of the numbers in order, each number in the form of the product of its prime factors*, only taking care to reject at each step those simple factors which have already appeared as factors of the L.C.M. Thus in the above example,

$$\text{L.C.M.} = 2 \mid \times 3 \mid \quad \mid \times 2 \mid \times 3 \mid \times 5 \mid \times 5 \mid \times 3 \mid \times 2 \times 2 \times 2 \mid \quad \mid$$
$$= 21600.$$

We have inserted cross lines, not because they are necessary, but to mark the course of the reasoning. Thus in the above L.C.M., it will be seen that we write down the first number 2 and the next number 3; the simple factors of the next number 6 are 2 and 3, both of which we reject because we have already written them down.

Of the next number 12 we only write down the simple factor 2, because we have already accepted the other two factors 2 and 3. So for the next numbers 18, 20, 25, 27. Of the number 32, we want only three of its five factors 2, as we already have two. Of the last number 45, we want no simple factors as we have used all of them, viz. 2, 3, 5, already. For the sake of practice, we will work out the L.C.M. again, taking the figures this time in an opposite order, namely, from the end to the beginning.

* By a "factor" of a number is meant a number that will exactly divide the number in question. Thus 2, 3, 4, 6, are factors of 12. Also 1 and 12 are called factors of 12.

By a prime number is meant a number that has no factors but itself and unity. Thus 2, 3, 5, 7, 11, 41, are prime numbers.

L.C.M. $= 3 \times 3 \times 5 \mid \times 2 \times 2 \times 2 \times 2 \times 2 \mid \times 3 \mid \times 5 \mid \; \mid \; \mid \; \mid$
$\mid \; \mid \; \mid = 21600.$

FRACTIONS.

7. *Definition.* The fraction of a quantity is a part of that quantity. Its size is determined by finding into how many equal parts the quantity must be divided, and how many of those parts must be taken, to obtain the part in question. Thus if we divide an apple into five equal parts and take three of them, we shall obtain a certain fraction of the apple. It is called three-fifths, and written $\frac{3}{5}$.

The lower of these numbers is called the Denominator (Nomen = name), and shews into how many equal parts the whole is divided, (and therefore the *size* of the equal parts); the upper is called the Numerator (Numerus = number), and shews how many of these parts are taken to form the fraction.

In fact, in three-fifths $\left(\frac{3}{5}\right)$, three is the *number*, and fifth the *name* of the equal parts taken. So in 7 shillings, seven is the number, (and might be called the numerator), shilling is the name, (and might be called the denominator).

From the definition above given, it is clear that the numerator must be less than the denominator. As however in arithmetic we also use the word fraction in a somewhat different sense, and in such a case the numerator is equal to or greater than the denominator, the fraction as above defined is called a *proper fraction*. If however we allow ourselves to make use of a fraction whose numerator is equal to or greater than its denominator, it is called an improper fraction, for it is clear that if the numerator is equal to the denominator, we are really not taking a *part* of the quantity,

but the *whole* quantity. Again if the numerator is greater than the denominator $\left(\text{e.g. } \frac{9}{7}\right)$ it is clear that the expression "the fraction of a quantity" is a confusion in terms (for we could not divide a quantity into 7 parts and take 9 of them). In such a case we must suppose any number of quantities, divide them each into the required number of parts and take the required number of those parts.

From this it is manifest that an improper fraction is equal to (if the numerator = the denominator) or is greater than (if the numerator is greater than the denominator) the whole quantity.

N.B. We have here slightly departed from the language of Arithmetical treatises, which call a fraction whose numerator is equal to its denominator a proper fraction.

Another definition of a fraction as arising from the first, is sometimes added, viz., that it equals the numerator divided by the denominator. We do not think the proof is quite satisfactory without generalizing the meaning of our words in a manner not fitting to pure arithmetic, nor do we think this the right place either to prove or adopt another definition. We shall return to this in the articles on Division of Fractions, merely stating that it will not be necessary for us to use this second definition till then.

8. *To reduce a mixed number to an improper fraction, and conversely.*

Ex. $3\frac{5}{7} = \frac{3 \times 7 + 5}{7} = \frac{21 + 5}{7} = \frac{26}{7}$.

For 1 (= 7 sevenths) $= \frac{7}{7}$, therefore 3 (= 21 sevenths) $= \frac{21}{7}$,

therefore $3\frac{5}{7} = \frac{21 + 5}{7} = \frac{26}{7}$.

So on the other hand $\frac{39}{5} = 7\frac{4}{5}$.

$$\begin{array}{l} 5)\,39\,(7 \\ \quad\underline{35} \\ \quad\ \ 4 \end{array}$$

For $\frac{5}{5} = 1$, therefore $\frac{35}{5} = 7$, therefore $\frac{39}{5} = 7\frac{4}{5}$.

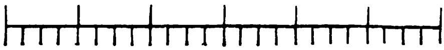
9. *To multiply a fraction by any whole number* or *integer.*

Rule. Either multiply the numerator or divide the denominator by it.

Thus $\frac{3}{7} \times 5 = \frac{15}{7}$; $\frac{5}{24} \times 4 = \frac{5}{6}$.

The first rule is clear, for as 3 apples multiplied by 5 is 15 apples, so 3 sevenths $\left(\frac{3}{7}\right)$ multiplied by 5 is 15 sevenths $\left(\frac{15}{7}\right)$.

The next rule may be explained thus: we may multiply $\frac{5}{24}$ by 4, either by taking four times as many twenty-fourths, or by taking the 5 parts each four times as large.



But, as we see in the figure, a sixth is four times as large as a twenty-fourth, (for we may divide the 24 parts into six groups of four parts), therefore $\frac{5}{24} \times 4 = \frac{5}{6}$.

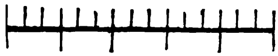
10. *To divide a fraction by any integer.*

Rule. Either divide the numerator or multiply the denominator by it.

Thus, $\frac{10}{7} \div 2 = \frac{5}{7}$; $\frac{2}{5} \div 3 = \frac{2}{15}$.

The first rule is clear, for as 10 apples divided by 2 is 5 apples, so 10 sevenths $\left(\frac{10}{7}\right)$ divided by 2 is 5 sevenths $\left(\frac{5}{7}\right)$.

The next rule may be explained thus: we can divide $\frac{2}{5}$ by 3, if we take the two parts each a third of their original size; but a third of a fifth, as we see in the figure,



is a fifteenth, therefore $\frac{2}{5} \div 3 = \frac{2}{15}$.

11. *To reduce a fraction to lower terms.*

Rule. Divide both numerator and denominator by the same number.

Thus, $\frac{6}{15} = \frac{2}{5}$.

The reason of this rule might easily be gathered from the second part of Art. 9. We will however prove it independently.

$\frac{6}{15}$ of a quantity is obtained by dividing it into 15 parts, and taking six of them. But we shall obtain the same result if we take only a third of the number of parts when each of the parts is three times as large. But a third of the number of parts gives us 2 parts. And if the parts are three times as large (see figure) they will be fifths,

for we may divide the 15 parts into 5 groups of 3 parts each. Therefore $\frac{6}{15} = \frac{2}{5}$.

Ex. $\frac{22176}{23328} = \frac{11088}{11664} = \frac{5544}{5832} = \frac{2772}{2916} = \frac{1386}{1458}$

$$= \frac{693}{729} = \frac{231}{243} = \frac{77}{81}.$$

Practical Rule. Divide numerator and denominator by 2 as often as possible, then by 3 as often as possible, similarly by 5, by 7, and by 11. Find the G.C.M. only as a last resort.

Rules are given for finding when a number is divisible by 2, 3, 5, and 11 in Art. 44.

N.B. If the numerator and denominator of a fraction be separated into such factors as can easily be seen by inspection, we may in almost all cases avoid finding the G.C.M. This is perhaps not a good method to teach those who are beginning fractions, but it will be found the shortest means in the end. It is an excellent method of detecting errors and checking work. Numerous rules may be given for accelerating the work of splitting a number into its prime factors.

12. *Division of concrete numbers by integers.*

Ex. 1. Divide £9. 7*s*. $3\frac{1}{4}$*d*. by 4.

	£.	*s*.	*d*.
4	9	7	$3\frac{1}{4}$
	2	6	$9\frac{13}{16}$

$$3\frac{1}{4} \div 4 = \frac{13}{4} \div 4$$
$$= \frac{13}{16}.$$

Ex. 2. Divide £7. 8*s.* $1\frac{5}{11}$*d.* by 6.

$$\begin{array}{r|rrr} & £. & s. & d. \\ 6 & 7 & 8 & 1\frac{5}{11} \\ \hline & 1 & 4 & 8\frac{8}{33} \end{array} \qquad 1\frac{5}{11} \div 6 = \frac{16}{11} \div 6$$

$$= \frac{16}{66} = \frac{8}{33}.$$

13. *To reduce a compound fraction to a simple one.*

Def. A compound fraction is the fraction of a fraction.

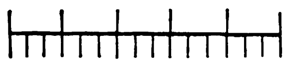
Rule. Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Thus, $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.

For $\frac{1}{3}$ of $\frac{4}{5}$ is the same as $\frac{4}{5} \div 3$, and is therefore equal to $\frac{4}{15}$ by Art. 10. Therefore $\frac{2}{3}$ of $\frac{4}{5}$, which must be twice as great, will equal $\frac{8}{15}$.

Or we may prove it independently thus:

In order to obtain $\frac{2}{3}$ of $\frac{4}{5}$, we must divide each of our four fifths into 3 parts, and take two of them. But if we



divide a fifth into three parts, each resultant part (see figure) is a fifteenth. Taking then two of these parts we get $\frac{2}{15}$.

As then each fifth gives us $\frac{2}{15}$, $\frac{4}{5}$ will give us $\frac{8}{15}$. Therefore $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.

$$\text{Ex.}\quad 3\frac{5}{9} \text{ of } 2\frac{1}{4} \text{ of } \frac{3}{28} \text{ of } 4\frac{1}{5} = \frac{\overset{2}{\cancel{\overset{8}{\cancel{32}}}}}{\cancel{9}} \text{ of } \frac{\cancel{9}}{\cancel{4}} \text{ of } \frac{3}{\underset{\cancel{4}}{\cancel{28}}} \text{ of } \frac{\overset{3}{\cancel{21}}}{5}$$

$$= \frac{18}{5}$$

$$= 3\frac{3}{5}. \quad \text{Ans.}$$

"Cancelling," which we have adopted above, is nothing but another name for reducing a fraction to its lowest terms, and consequently the rule for cancelling is to divide numerator and denominator by the same numbers.

13′. *To reduce fractions to their least common denominator.*

We have proved that we may *divide* the numerator and denominator of a fraction by the same number, without altering its value; and it is clear that we may *multiply* the numerator and denominator of a fraction by the same number, without altering its value. Thus, $\frac{6}{9} = \frac{2}{3}$, and also $\frac{2}{3} = \frac{6}{9}$.

Suppose we wish to reduce the fraction $\frac{4}{7}$ to the denominator 42. Since the denominator 42 is derived from the denominator 7 by multiplying 7 by 6 ($42 \div 7 = 6$), the new numerator will be derived from the old numerator 4, by multiplying it by 6. Therefore $\frac{4}{7} = \frac{24}{42}$.

Hence we get the rule for reducing fractions to their least common denominator.

RULE. Find the L.C.M. of all the denominators (as that is the *smallest* number that *all* the denominators will divide exactly), and take this for the common denominator; for the new numerators, multiply each numerator by the quotient obtained by dividing the common denominator by its own denominator (as these quotients will be the numbers by which we have multiplied the respective denominators).

The object of this rule is to add together, or to subtract from each other, fractions of different denominators. We know that 3 apples + 5 apples = 8 apples; 3 pears + 5 pears = 8 pears; but we do not know how to express more simply 3 apples + 5 pears, because these are of different denominations. If, however, we were told that 3 apples were equal to 15 nuts, and 5 pears to 20 nuts, we should then know that 3 apples + 5 pears would be equal to 35 nuts, because we have now reduced the original denominations to the same denomination. So in fractions, we know that $\frac{3}{7}+\frac{5}{7}$ (3 sevenths + 5 sevenths) $=\frac{8}{7}$, and that $\frac{3}{9}+\frac{5}{9}=\frac{8}{9}$; but we do not at once know what $\frac{3}{7}+\frac{5}{9}$ are, because these are of different denominations. But by the above rule,

$$\frac{3}{7}=\frac{3\times 9}{63}=\frac{27}{63};\quad \frac{5}{9}=\frac{7\times 5}{63}=\frac{35}{63};$$

therefore $\frac{3}{7}+\frac{5}{9}=\frac{27}{63}+\frac{35}{63}=\frac{62}{63}$, because we have now reduced the original denominations to the same denomination.

Similar remarks may evidently be applied to subtraction.

14. *Addition of fractions.*

Ex. Simplify $2\frac{3}{4}$ of $3\frac{2}{3}+\frac{111}{16}+2\frac{4}{5}$ of $4\frac{1}{8}$ of $1\frac{3}{8}+4\frac{2}{3}$ of $\frac{2}{15}$ of $2\frac{1}{8}$ of $1\frac{3}{7}$.

$$2\frac{3}{4} \text{ of } 3\frac{2}{3}=\frac{11}{4} \text{ of } \frac{11}{3}=\frac{121}{12}=10\frac{1}{12}.$$

$$\frac{111}{16}=5\frac{15}{16}.$$

$$2\frac{4}{5} \text{ of } 4\frac{1}{8} \text{ of } 1\frac{3}{8}=\frac{\overset{7}{\cancel{14}}}{5} \text{ of } \frac{33}{\underset{4}{\cancel{8}}} \text{ of } \frac{11}{8}=\frac{2541}{160}=15\frac{141}{160}.$$

$$4\frac{2}{3} \text{ of } \frac{2}{15} \text{ of } 2\frac{1}{8} \text{ of } 1\frac{3}{7}=\frac{\overset{\cancel{7}}{\cancel{14}}}{3} \text{ of } \frac{\cancel{2}}{\underset{3}{\cancel{15}}} \text{ of } \frac{17}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{8}}} \text{ of } \frac{\overset{\cancel{2}}{\cancel{10}}}{\cancel{7}}=\frac{17}{9}=1\frac{8}{9}.$$

$$\therefore \text{ Ans.}=10\frac{1}{12}+5\frac{15}{16}+15\frac{141}{160}+1\frac{8}{9}$$

$$=31\frac{120+1350+1269+1280}{1440}$$

$$=31\frac{4019}{1440}$$

$$=33\frac{1139}{1440}.$$

N.B. If it is preferred, the following method of doing the above example may be adopted.

$$2\frac{3}{4} \text{ of } 3\frac{2}{3} + \frac{111}{16} + 2\frac{4}{5} \text{ of } 4\frac{1}{8} \text{ of } 1\frac{3}{8} + 4\frac{2}{3} \text{ of } \frac{2}{15} \text{ of } 2\frac{1}{8} \text{ of } 1\frac{3}{7}$$

$$= \frac{11}{4} \text{ of } \frac{11}{3} + \frac{111}{16} + \frac{\overset{7}{\cancel{14}}}{5} \text{ of } \frac{33}{\underset{4}{\cancel{8}}} \text{ of } \frac{11}{8} + \frac{\overset{\cancel{2}}{\cancel{14}}}{3} \text{ of } \frac{\cancel{2}}{\underset{3}{\cancel{15}}} \text{ of } \frac{17}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{8}}} \text{ of } \frac{\overset{5}{\cancel{10}}}{\cancel{7}}$$

$$= \frac{121}{12} + \frac{111}{16} + \frac{2541}{160} + \frac{17}{9}$$

$$= 10\frac{1}{12} + 5\frac{15}{16} + 15\frac{141}{160} + 1\frac{8}{9}$$

$$= 31\,\frac{120 + 1350 + 1269 + 1280}{1440}$$

$$= 31\frac{4019}{1440}$$

$$= 33\frac{1139}{1440}.$$

The same method might be applied in Ex. 3, Art. 15, instead of the one there employed, and in other Arts.

15. *Subtraction of fractions.*

Always work with mixed numbers as in Addition. When this is not possible, proceed as follows:

Ex. 1. $7\frac{1}{4} - 2\frac{5}{6} = 5\frac{3-10}{12} = 4\frac{15-10}{12} = 4\frac{5}{12}$. Ans.

Ex. 2. $9 - 2\frac{3}{5} = 7 - \frac{3}{5} = 6\frac{5-3}{5} = 6\frac{2}{5}$. Ans.

In cases of many additions and subtractions in the same sum, *never insert brackets.* They are useless. The best rule is; add the fractions or numbers that have to be

added, and add those that have to be subtracted; then subtract the latter result from the former.

Ex. 3. Simplify

$$5\frac{1}{5} - 2\frac{5}{6} - 3\frac{3}{10} + \frac{13}{2} - 16\frac{1}{4} + 3\frac{1}{12} + 8\frac{1}{8}.$$

$$\begin{aligned} 5\frac{1}{5} + \frac{13}{2} + 3\frac{1}{12} + 8\frac{1}{8} &= 5\frac{1}{5} + 6\frac{1}{2} + 3\frac{1}{12} + 8\frac{1}{8} \\ &= 22\,\frac{24 + 60 + 10 + 15}{120} \\ &= 22\,\frac{109}{120}; \end{aligned}$$

$$\begin{aligned} 2\frac{5}{6} + 3\frac{3}{10} + 16\frac{1}{4} &= 21\,\frac{50 + 18 + 15}{60} \\ &= 21\,\frac{83}{60} \\ &= 22\,\frac{23}{60}; \end{aligned}$$

$$\begin{aligned} \therefore \text{Ans.} = 22\,\frac{109}{120} - 22\,\frac{23}{60} &= \frac{109 - 46}{120} \\ &= \frac{63}{120} \\ &= \frac{21}{40}. \end{aligned}$$

16. *Multiplication of fractions.* See Note A, and the next article.

17. *Division of fractions.* See Note B.

The whole theory of multiplication and division may

be dealt with and illustrated in many different manners, more or less simple. Some of these will be discussed in the notes at the end of the volume. Whatever plan of explanation be finally followed, it is clear, however, that for beginners, the theory should be made as short and simple as possible, and all complex, or even exact reasoning, on a subject so confessedly difficult, be left, till they have attained a much riper age. The following would seem to be sufficient at first.

DEF. *If we multiply the numerators of two fractions together for a new numerator, and the denominators together for a new denominator, we are said to multiply the fractions together.*

Since division reverses multiplication, the rule for division of fractions will be *to invert the divisor, and multiply.*

18. Since by the above rule of division of fractions

$$5 \div 7 = \frac{5}{1} \div \frac{7}{1} = \frac{5}{1} \times \frac{1}{7} = \frac{5}{7},$$

we get a new definition of a fraction, namely, *the numerator divided by the denominator.*

19. *To find the value of a fraction of a concrete quantity.*

Some latitude of working should be allowed here. However, a very general rule can be given *not to reduce to lowest denominations.* This is almost always a tedious and unnecessary method. We will give a few examples.

Ex. 1. Find the value of £7. 9*s.* $8\frac{3}{4}$*d.* $\times 3\frac{2}{5}$.

$$\begin{array}{rrl} \pounds. & s. & d. \\ 7 & 9 & 8\frac{3}{4} \\ & & 3 \\ \hline 22 & 9 & 2\frac{1}{4} \\ 2 & 19 & 10\frac{7}{10} \\ \hline 25 & 9 & 0\frac{19}{20} \end{array} \text{ Ans.} \qquad\qquad \begin{array}{r|rrl} & \pounds. & s. & d. \\ & 7 & 9 & 8\frac{3}{4} \\ & & & 2 \\ \hline 5 & 14 & 19 & 5\frac{1}{2} \\ \hline & 2 & 19 & 10\frac{7}{10} \end{array}$$

Ex. 2. Find the value of £7. 9*s*. $8\frac{2}{3}d. \div 5\frac{3}{5}$.

$$\begin{array}{r|rrl} & \pounds. & s. & d. \\ & 7 & 9 & 8\frac{2}{3} \div 5\frac{3}{5},\ \text{i.e.} \div \frac{28}{5},\ \text{i.e.} \times \frac{5}{28}. \\ & & & 5 \\ \hline 28 & 37 & 8 & 7\frac{1}{3} \\ \hline & 1 & 6 & 8\frac{5}{6} \end{array} \text{ Ans.}$$

$$23\frac{1}{3} \div 28 = \frac{\cancelto{5}{\cancelto{10}{70}}}{3} \times \frac{1}{\cancelto{2}{\cancelto{4}{28}}}$$

$$= \frac{5}{6}.$$

Ex. 3. Find the value of £7. 9*s*. $8\frac{3}{4}d. \div \frac{7}{26}$.

$$\begin{array}{rrl} \pounds. & s. & d. \\ 7 & 9 & 8\frac{3}{4} \div \frac{7}{26},\ \text{i.e.} \times \frac{26}{7},\ \text{i.e.} \times 3\frac{5}{7}. \end{array}$$

$$\begin{array}{rrl} \pounds. & s. & d. \\ 7 & 9 & 8\frac{3}{4} \\ & & 3 \\ \hline 22 & 9 & 2\frac{1}{4} \\ 5 & 6 & 11\frac{11}{28} \\ \hline 27 & 16 & 1\frac{9}{14} \end{array} \text{ Ans.} \qquad\qquad \begin{array}{r|rrl} & \pounds. & s. & d. \\ & 7 & 9 & 8\frac{3}{4} \\ & & & 5 \\ \hline 7 & 37 & 8 & 7\frac{3}{4} \\ \hline & 5 & 6 & 11\frac{11}{28} \end{array}$$

Ex. 4. Find the value of $\dfrac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\dfrac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\dfrac{77}{540}$ of 27*s*.

$$\frac{3\frac{7}{11}}{4\frac{2}{7}} \text{ of } \frac{10\frac{5}{7}}{7\frac{1}{2}} \text{ of } \frac{77}{540} \text{ of } 27s.$$

$$= \frac{\overset{4}{\cancel{40}}}{\cancel{11}} \times \frac{\cancel{7}}{\underset{3}{\cancel{30}}} \text{ of } \frac{\overset{5}{\cancel{75}}}{\cancel{7}} \times \frac{2}{\cancel{15}} \text{ of } \frac{\overset{7}{\cancel{77}}}{\underset{\underset{\underset{3}{\cancel{12}}}{\cancel{60}}}{\cancel{540}}} \text{ of } \frac{\overset{9}{\cancel{27}}}{1} s.$$

$$= \frac{14}{3} s.$$

$$= 4\frac{2}{3} s.$$

$$= 4s.\ 8d. \quad \text{Ans.}$$

20. *To reduce one quantity (or the fraction of one quantity) to the fraction of another (or of the fraction of another).*

Ex. 1. Reduce 3 cwt. 2 qrs. 3 lbs. to the fraction of a ton.

cwt.	qrs.	lbs.		ton.
3	2	3		1
4				20
14 qrs.				20 cwt.
28				112
115				2240 lbs.
28				
395 lbs.				

$$\therefore 3 \text{ cwt. } 2 \text{ qrs. } 3 \text{ lbs.} = \frac{395}{2240} \text{ of } 1 \text{ ton}$$

$$= \frac{79}{448} \text{ of } 1 \text{ ton.} \quad \text{Ans.}$$

The reason of this is clear. For, since 1 lb. is $\frac{1}{2240}$ of 2240 lbs., therefore 395 lbs. is $\frac{395}{2240}$ of 2240 lbs., i.e. 3 cwt. 2 qrs. 3 lbs. is $\frac{395}{2240}\left(\text{or } \frac{79}{448}\right)$ of 1 ton.

Ex. 2. Reduce $\frac{3}{8}$ of $1\frac{1}{2}$ of 10*s.* $7\frac{1}{2}$*d.* to the fraction of £4. 4*s.* $4\frac{1}{2}$*d.*

s. *d.*	£. *s.* *d.*
10 $7\frac{1}{2}$	4 4 $4\frac{1}{2}$
12	20
127*d.*	84*s.*
2	12
255 half-pence.	1012*d.*
	2
	2025 half-pence.

$$\therefore \frac{3}{8} \text{ of } 1\frac{1}{2} \text{ of } 10s.\ 7\tfrac{1}{2}d. = \frac{3}{8} \text{ of } 1\frac{1}{2} \text{ of } \frac{255}{2025} \text{ of } £4.\ 4s.\ 4\tfrac{1}{2}d.$$

$$= \frac{\cancel{3}}{8} \text{ of } \frac{\cancel{3}}{2} \text{ of } \frac{\overset{17}{\cancel{\overset{51}{\cancel{255}}}}}{\underset{15}{\cancel{\underset{45}{\cancel{\underset{135}{\cancel{\underset{405}{\cancel{2025}}}}}}}}} \text{ of } £4.\ 4s.\ 4\tfrac{1}{2}d.$$

$$= \frac{17}{240} \text{ of } £4.\ 4s.\ 4\tfrac{1}{2}d. \quad \text{Ans.}$$

Here again, since 10*s.* $7\frac{1}{2}$*d.* = $\frac{255}{2025}$ of £4. 4*s.* $4\frac{1}{2}$*d.*, (by the 1st Ex.), therefore $\frac{3}{8}$ of $1\frac{1}{2}$ of 10*s.* $7\frac{1}{2}$*d.* = $\frac{3}{8}$ of $1\frac{1}{2}$

of $\frac{255}{2025}$ of £4. 4*s.* $4\frac{1}{2}$*d.*, which, worked out as above, gives us the result $\frac{17}{240}$ of £4. 4*s.* $4\frac{1}{2}$*d.*

Ex. 3. Reduce $5\frac{4}{5}$ of £2. 2*s.* 6*d.* to the fraction of $5\frac{1}{3}$ of £1. 9*s.*

$$\begin{array}{lll} £. & s. & d. \\ 2 & 2 & 6 \\ 20 & & \\ \hline 42s. & & \\ 2 & & \\ \hline \multicolumn{3}{l}{85 \text{ sixpences.}} \end{array} \qquad\qquad \begin{array}{ll} £. & s. \\ 1 & 9 \\ 20 & \\ \hline 29s. & \\ 2 & \\ \hline \multicolumn{2}{l}{58 \text{ sixpences.}} \end{array}$$

$$\therefore\ 5\frac{4}{5} \text{ of } £2.\ 2s.\ 6d. = 5\frac{4}{5} \text{ of } \frac{85}{5\frac{1}{3} \text{ of } 58} \text{ of } \left(5\frac{1}{3} \text{ of } £1.\ 9s.\right)$$

$$= \frac{\cancel{29}}{\cancel{5}} \text{ of } \frac{\overset{17}{\cancel{85}}}{\underset{2}{\cancel{58}}} \times \frac{3}{16} \text{ of } \left(5\frac{1}{3} \text{ of } £1.\ 9s.\right)$$

$$= \frac{51}{32} \text{ of } \left(5\frac{1}{3} \text{ of } £1.\ 9s.\right)$$

$$= 1\frac{19}{32} \text{ of } \left(5\frac{1}{3} \text{ of } £1.\ 9s.\right). \quad \text{Ans.}$$

Ex. 4. Reduce $\frac{7}{9}$ of £1 $-\frac{2}{5}$ of 21*s.* to the fraction of 10*s.* 6*d.*

$$\frac{7}{9} \text{ of } £1 - \frac{2}{5} \text{ of } 21s. = \frac{\frac{7}{9} \text{ of } 20 - \frac{2}{5} \text{ of } 21}{10\frac{1}{2}} \text{ of } 10s.\ 6d.$$

$$\text{Now } \frac{\frac{7}{9}\text{ of }20-\frac{2}{5}\text{ of }21}{10\frac{1}{2}}=\frac{\frac{140}{9}-\frac{42}{5}}{10\frac{1}{2}}$$

$$=\frac{15\frac{5}{9}-8\frac{2}{5}}{10\frac{1}{2}}$$

$$=\frac{7\frac{25-18}{45}}{10\frac{1}{2}}$$

$$=\frac{7\frac{7}{45}}{10\frac{1}{2}}$$

$$=\frac{\overset{46}{\cancel{322}}}{45}\times\frac{2}{\underset{3}{\cancel{21}}}$$

$$=\frac{92}{135};$$

$$\therefore \frac{7}{9}\text{ of £1}-\frac{2}{5}\text{ of }21s.=\frac{92}{135}\text{ of }10s.\ 6d.\quad\text{Ans.}$$

21. *Rule of precedence of the four simple rules in Arithmetical questions (with or without brackets).*

Multiplication and division precede addition and subtraction.

If any of the quantities are in brackets, of course each bracket (being a single quantity with respect to the rest

of the expression) must be simplified first, before the above rule applies to the whole quantity to be simplified.

Ex. 1. $1 + 2 \times 3 + 4 = 1 + 6 + 4 = 11.$

Ex. 2. $(1 + 2) \times 3 + 4 = 3 \times 3 + 4 = 9 + 4 = 13.$

Ex. 3. $1 + 2 \times (3 + 4) = 1 + 2 \times 7 = 1 + 14 = 15.$

Ex. 4. $(1 + 2) \times (3 + 4) = 3 \times 7 = 21.$

DECIMALS.

22. By means of the radix ten, we are able to express any integers from unity *upwards* without limit. By an extension of the method adopted in numeration and notation, we are able to get a series of "decimal fractions" from unity *downwards* without limit. The series of decimal fractions must have ten, or some power* of ten, for their denominators. The turning-points in this extension of the old system are as follows:

....		
thousands		
hundreds............	$1000 \div 10 =$	$100,$
tens..................	$100 \div 10 =$	$10,$
units	$10 \div 10 =$	$1,$
tenths	$1 \div 10 =$	$\frac{1}{10},$
hundredths.........	$\frac{1}{10} \div 10 =$	$\frac{1}{100},$
thousandths	$\frac{1}{100} \div 10 =$	$\frac{1}{1000},$
....		

* See Note to Art. 27.

Now in the old system for integers, in such a number as 1234, since that system commenced from a fixed point, unity, we know that the 4 of the above number represents units, the 3, tens, the 2, hundreds, and the 1, a thousand, but since the extension of the system for fractions does not begin at a fixed point, nor end at a fixed point, we must know the value of some particular digit, as unity, in order to know the real position and value of the rest. This is done by placing a point after the unit, commonly called a decimal point, as thus; 123·4567. Here then the 3 represents units, the 2, tens, the 1, a hundred; whereas, on the other hand, the 4 represents tenths, the 5, hundredths, the 6, thousandths, the 7, ten-thousandths (or tenths of thousandths). We may tabulate these results in the following manner.

A figure in the first place before the decimal point represents units.

A figure in the second place before the decimal point represents tens, (unity × 10).

A figure in the third place before the decimal point represents hundreds, (ten × 10).

A figure in the fourth place before the decimal point represents thousands, (hundred × 10).

....

Again :—

A figure in the first place after the decimal point represents tenths, (unity ÷ 10).

A figure in the second place after the decimal point represents hundredths, (tenth ÷ 10).

A figure in the third place after the decimal point represents thousandths, (hundredth ÷ 10).

A figure in the fourth place after the decimal point represents ten-thousandths, (thousandth $\div$ 10).

....

Thus the number 123·4567 would be read, one hundred and twenty three, decimal 4567; where "decimal 4567" would mean 4 tenths, 5 hundredths, 6 thousandths, and 7 ten-thousandths.

Again, the number ·007 would be read, decimal 007, and would mean 7 thousandths, i.e., $\frac{7}{1000}$.

Conversely, $\frac{7}{100} = {}^{\cdot}07$.

23. From the above article, the *rules for reducing a decimal to a fraction, and a fraction with some power of ten for its denominator to a decimal*, may be at once deduced.

$$21{\cdot}0347 = 20 + 1 + \frac{3}{100} + \frac{4}{1000} + \frac{7}{10000} = 21 + \frac{300 + 40 + 7}{10000}$$
$$= 21\frac{347}{10000}.$$

Conversely,

$$\frac{10051}{10000} = 1 + \frac{50}{10000} + \frac{1}{10000} = 1 + \frac{5}{1000} + \frac{1}{10000} = 1{\cdot}0051.$$

Again, $11\frac{5}{1000} = 11{\cdot}005$.

Hence we are in a position to give the verbal results.

To reduce a decimal to a fraction.

RULE. Write down the figures after the decimal point, for the numerator, and 1 followed by as many cyphers as there are decimal places, for the denominator.

To reduce a fraction, whose denominator is some power of ten, to a decimal fraction.

RULE. Write down the numerator of the fraction, and mark off as many decimal places as there are cyphers in the denominator.

24. *Any decimal is multiplied by* 10, 100, 1000, *&c. by moving the decimal point one, two, three, &c. decimal places to the right.*

Any decimal is divided by 10, 100, 1000, *&c. by moving the decimal point one, two, three, &c. decimal places to the left.*

Thus, $\cdot 012 \times 100 = \frac{12}{1000} \times \frac{100}{1} = \frac{12}{10} = 1\cdot 2.$

And $1\cdot 234 \div 100 = 1\frac{234}{1000} \div 100 = \frac{1234}{1000} \times \frac{1}{100}$

$= \frac{1234}{100000} = \cdot 01234.$

25. *Multiplication of Decimals.*

Leave out all superfluous decimal cyphers, thus:—

Ex. Multiply together ·0003, 12·5, 50000, 20, ·0004.

$$\begin{array}{r}
\cdot 0003 \\
12\cdot 5 \\
\hline
\cdot 00375 \\
50000 \\
\hline
187\cdot 50000 \\
20 \\
\hline
3750\cdot 0 \\
\cdot 0004 \\
\hline
1\cdot 5000
\end{array}$$

Ans. = 1·5.

26. *Division of Decimals.*

It is as well that there should be but one rule learnt for the division of decimals. Perhaps the following will be found to be the clearest and the easiest to apply.

If the given divisor is not a whole number, make it so by removing its decimal point altogether, and shift the decimal point of the dividend as many places to the right as there were decimal figures in the divisor; annexing for this purpose, if necessary, decimal cyphers to the dividend.

Then, divide as if the given decimals were common integers; and when, in the process of division, the decimal point of the dividend is arrived at, place a decimal point in the quotient.

Decimal cyphers may be annexed to the dividend to any extent that may be required for carrying on the division.

Ex. Divide ·04 by 20, ·4 by ·002, ·0004 by ·0002, ·004 by ·2, 400 by ·02.

27. *To reduce a fraction to a decimal.*

RULE. Divide the numerator by the denominator.

This follows from the 2nd definition of a vulgar fraction. Art. 18.

If the denominator of the fraction reduced to its lowest terms, consists only of powers* of two and five as factors,

* By the "power" of a number is meant the product of the number into itself any number of times. Thus 10 is said to be the first power

the decimal will (i) terminate. If it has no factors of two or five, the decimal will be (ii) a pure circulator. If it has factors of two or five, or both, *with other factors*, the decimal will be (iii) a mixed circulator.

For (i), leaving out of consideration the decimal point, a fraction is reduced to a decimal by multiplying the numerator by any power of 10 (i.e. by any powers of 2 and 5), and dividing the result by the denominator. Now, with each power of ten in the numerator, a factor of 2 and 5 will disappear (by cancelling) in the denominator. Also, when the less power of either 2 or 5 is exhausted, there will remain a power of 5 or 2. Each factor of this power will disappear by means of a further power of ten in the numerator. Hence, if the denominator of a fraction reduced to its lowest terms consists only of powers of two and five as factors, it is clear that the decimal will terminate.

It is also clear that the number of decimal places in the decimal will equal the number of factors of 2 or 5 in the denominator, whichever happens to be the greater. For every power of 10 introduced as a factor of the numerator gives one decimal place.

Again (ii), if the denominator has neither 2 nor 5 as a factor, it is clear that the decimal *will not terminate.* For none of the factors of the denominator can either disappear with those of the numerator (for the fraction is in its lowest terms) or with the powers of 2 and 5 (i.e. of 10) by which the numerator is multiplied. We have now to prove that the decimal will *circulate.*

In the division, we always bring down the same figure, namely, a cypher; hence, if ever at any step of the division

of 10, 100 the second power of 10 (because it equals 10 × 10), 1000 the third power of 10 (10 × 10 × 10), &c.

an old remainder should be repeated, from that step the whole process will be repeated. But, since a remainder must be less than the divisor, the number of different remainders is limited. Therefore either the decimal will recur or will terminate. But we have proved that it does not terminate. Therefore it recurs.

Also it is clear from the above, that the number of decimal places in the period *at most* is one less than the denominator.

Lastly (iii), if the denominator of the fraction reduced to its lowest terms has factors of 2 or 5, or both, as well as other factors, the decimal will be a mixed circulator, the number of places before the period being equal to the power of 2 or 5 in the denominator, whichever happens to be greater, and the number of figures in the period being *at most* one less than the denominator when deprived of its factors of 2 and 5.

The truth of the last statement will be best seen from a few instances.

$$\frac{5}{6}\left(=\frac{5}{2\times 3}\right)=\frac{50}{6}\div 10=\frac{25}{3}\div 10=8\cdot\dot{3}\div 10=\cdot 8\dot{3}.$$

$$\frac{5}{44}\left(=\frac{5}{4\times 11}\right)=\frac{500}{44}\div 100=\frac{125}{11}\div 100=11\cdot\dot{3}\dot{6}\div 100=\cdot 11\dot{3}\dot{6}.$$

$$\frac{371}{1125}\left(=\frac{371}{125\times 9}\right)=\frac{371000}{1125}\div 1000=\frac{2968}{9}\div 1000$$
$$=329\cdot\dot{7}\div 1000=\cdot 329\dot{7}.$$

$$\frac{4}{35}\left(=\frac{4}{5\times 7}\right)=\frac{40}{35}\div 10=\frac{8}{7}\div 10=1\cdot\dot{1}4285\dot{7}\div 10=\cdot 1\dot{1}4285\dot{7}.$$

The object of the form given to the fractions in the brackets is to indicate the reason of selection of the ensuing powers of ten, by which we multiply numerator and denominator.

The last example gives us the period as large as it can be, viz. one less than the denominator when deprived of its factors of 2 and 5.

All the above results, with others of somewhat less importance, and greater difficulty of arithmetical proof, are tabulated in Art. 44.

28. *Proof of the rule for reducing a circulating decimal to a fraction.*

First, for a pure circulator, as $.\dot{3}\dot{7}$.

$$.\dot{3}\dot{7} \times 100 = 37.\dot{3}\dot{7}$$

$$= 37 + .\dot{3}\dot{7};$$

$$\therefore\ .\dot{3}\dot{7} \times 99 = 37;$$

$$\therefore\ .\dot{3}\dot{7} = \frac{37}{99}.$$

This agrees with the rule.

Next, for a mixed circulator, as $.3\dot{4}\dot{7}$.

$$.3\dot{4}\dot{7} \times 10 = 3.\dot{4}\dot{7}$$

$$= 3\frac{47}{99} \text{ (by the above)}$$

$$= \frac{3 \times 99 + 47}{99}$$

$$= \frac{3(100 - 1) + 47}{99}$$

$$= \frac{347 - 3}{99};$$

$$\therefore\ .3\dot{4}\dot{7} = \frac{347 - 3}{990}.$$

This also agrees with the rule.

29. $\dot{9} = \frac{9}{9} = 1, \quad \cdot 0\dot{9} = \frac{9}{90} = \frac{1}{10} = \cdot 1, \quad \cdot 00\dot{9} = \frac{9}{900}$

$$= \frac{1}{100} = \cdot 01, \text{ \&c.}$$

Hence, generally, the period 9 at any place of decimals may be replaced by a *one* of the preceding place. Thus we can assert at once that

$$\cdot 4\dot{9} = \cdot 5, \quad \cdot 74\dot{9} = \cdot 75, \quad \cdot 124\dot{9} = \cdot 125, \text{ \&c.}$$

30. It is well to remember that

$$1 \text{ farthing} = \frac{1}{4} = \cdot 25.$$

$$1 \text{ halfpenny} = \frac{1}{2} (= \cdot 50) = \cdot 5.$$

$$3 \text{ farthings} = \frac{3}{4} = \cdot 75.$$

And of course, conversely, $\cdot 25 = \frac{1}{4}, \quad \cdot 75 = \frac{3}{4}, \quad \cdot 5 = \frac{1}{2}.$

31. *To find the value of a decimal of any given quantity.*

It is well, in finding the value of a decimal of a given quantity, when the decimal is a circulator, *if the period is a long one, generally* to reduce to a fraction, if short, to work in decimals. We will give an example of the latter.

Ex. Find the value of $2 \cdot 1\dot{4}\dot{5}$ of 5*s.* $8\frac{3}{4}$*d.*

$$\begin{array}{r} 5s.\ 8\tfrac{3}{4}d. \\ 12 \\ \hline 68d. \\ 4 \\ \hline 275 \text{ farthings.} \end{array}$$

$$\begin{array}{r}
2\cdot14545 \\
\underline{275} \\
1072727 \\
1501818,1 \\
\underline{429090,90} \\
4\,|\underline{589\cdot99999} \\
12\,|\underline{147\cdot49999} \\
12-3
\end{array}$$

Ans. $= 12s.\ 3\cdot4\dot{9}d. = 12s.\ 3\cdot5d.$ (by Art. 29) $= 12s.\ 3\frac{1}{2}d.$*

On examining the above, it will be observed that the proper corrections are introduced throughout, arising from the recurrence of the period. This of course is necessary to ensure a right result. Observe also that the *whole result*, 589·999...... farthings is divided by 4, as we never give answers in decimals, or fractions, *of farthings.*

N.B. The answer above in farthings is $589\cdot\dot{9}$, i.e. 590 farthings $= 12s.\ 3\frac{1}{2}d.$ This is a rather shorter and better method than the one adopted. As however we wished to shew a *general* method, and this immediate simplification only applied to the particular case, we purposely avoided it. No other period but 9 could have been thus simplified.

32. It will be found convenient in working decimals to remember the following result:

$$\frac{1}{7} = \cdot\dot{1}4285\dot{7}.$$

Now the order of largeness of the above figures is 1, 2, 4, 5, 7, 8, and it will be found that if we begin with these figures respectively, and take the period in the same order, we shall get the values of the fractions

* The commas are introduced to point out where the process of multiplication *usually* stops.

$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ respectively. Thus,

$$\frac{2}{7} = \cdot\dot{2}8571\dot{4},$$

$$\frac{3}{7} = \cdot\dot{4}2857\dot{1},$$

$$\frac{4}{7} = \cdot\dot{5}7142\dot{8},$$

$$\frac{5}{7} = \cdot\dot{7}1428\dot{5},$$

$$\text{and } \frac{6}{7} = \cdot\dot{8}5714\dot{2}.$$

Conversely, any one of these decimals may at once be reduced back again to a fraction by considering the order of merit of the figures of its period. Thus, $\cdot\dot{5}7142\dot{8} = \frac{4}{7}$, because 5 is the *fourth largest figure* of the six figures of the period. This is not arbitrary, but follows from Arts. 27 and 44, since the period contains the greatest number of figures possible, viz. one less than the denominator.

Example:

$$\cdot\dot{2}8571\dot{4} \text{ of } £30 + 6\cdot\dot{8}5714\dot{2}£ + \cdot\dot{6} \text{ of } \cdot\dot{7}1428\dot{5}$$
$$\text{of } \cdot6£ + 1\cdot\dot{3} \text{ of } \cdot\dot{4}2857\dot{1}s.$$

$$= \frac{2}{7} \text{ of } £30 + 6\frac{6}{7}£ + \frac{2}{\cancel{3}} \text{ of } \frac{\cancel{5}}{7} \text{ of } \frac{\cancel{3}}{\cancel{5}}£ + 1\frac{1}{3} \text{ of } \frac{3}{7}s.$$

$$= 8\frac{4}{7}£ + 6\frac{6}{7}£ + \frac{2}{7}£ + \frac{4}{\cancel{3}} \text{ of } \frac{\cancel{3}}{7}s.$$

$$= 15\frac{5}{7}£ + \frac{4}{7}s.$$

$= £15.\ 14\frac{2}{7}s. + \frac{4}{7}s.$

$= £15.\ 14\frac{6}{7}s.$

$= £15.\ 14s.\ 10\frac{2}{7}d.$ Ans.

32′. Reduce to a decimal accurate to 5 places,

$$16\left(\frac{1}{5} - \frac{1}{3\cdot5^3} + \frac{1}{5\cdot5^5} - \frac{1}{7\cdot5^7} + \ldots\ldots\right) - \frac{4}{239}\ *.$$

$\frac{1}{5} = \cdot2,$

$\frac{1}{5^2} = \cdot04,$

$\frac{1}{5^3} = \cdot008;$ $\qquad\therefore\ \frac{1}{3\cdot5^3} = \cdot00266666.$

$\frac{1}{5^4} = \cdot0016,$

$\frac{1}{5^5} = \cdot00032,$

$\frac{1}{5^6} = \cdot000064,$

$\frac{1}{5^7} = \cdot0000128;$ $\qquad\therefore\ \frac{1}{7\cdot5^7} = \cdot00000182.$

$\frac{1}{5^8} = \cdot00000256,$

$\frac{1}{5^9} = \cdot000000512;$ $\qquad\therefore\ \frac{1}{9\cdot5^9} = \cdot00000005.$

....

$\frac{1}{5} = \cdot2,$

* The *dot* in the denominators implies multiplication. 5^3 denotes $5\times5\times5$; 5^5, $5\times5\times5\times5\times5$; and so on; so that 5^3 is the third power, 5^5, the fifth power, of 5, and so on.

$$\frac{1}{5\cdot5^5} = \cdot000064, \qquad \frac{1}{3\cdot5^3} = \cdot00266666.$$

$$\frac{1}{9\cdot5^9} = \cdot00000005, \qquad \frac{1}{7\cdot5^7} = \cdot00000182.$$

$$\therefore\ \frac{1}{5} + \frac{1}{5\cdot5^5} + \frac{1}{9\cdot5^9} = \cdot20006405\,; \quad \therefore\ \frac{1}{3\cdot5^3} + \frac{1}{7\cdot5^7} = \cdot00266848.$$

$$\therefore\ \frac{1}{5} - \frac{1}{3\cdot5^3} + \frac{1}{5\cdot5^5} - \frac{1}{7\cdot5^7} + \ldots = \cdot19739557\,;$$

$$\therefore\ 16\left(\frac{1}{5} - \frac{1}{3\cdot5^3} + \frac{1}{5\cdot5^5} - \frac{1}{7\cdot5^7} + \ldots\right) = 3\cdot15832912.$$

$$\frac{4}{239} = \cdot01673640\,;$$

$$\therefore\ 16\left(\frac{1}{5} - \frac{1}{3\cdot5^3} + \frac{1}{5\cdot5^5} - \frac{1}{7\cdot5^7} + \ldots\right) - \frac{4}{239} = 3\cdot14159272\,;$$

$$\therefore\ \text{Ans.} = 3\cdot14159.$$

Proportion.

33. Proportion is usually taught by the method of "statement." There are several advantages arising from the use of this method. It is the neatest, it is the shortest, it can be as easily taught as any other method *with respect to getting out right answers*, it is the method universally required from a business man, so that it would be a disgrace not to know how to "state a rule of three sum." There is also the "common sense" method which has this inestimable advantage, that those who use it understand the reason of the steps, which we cannot, we are afraid, generally predicate of those who use the statement method.

As, moreover, Arithmetic should appeal to the reason, and not merely to the memory, it would seem well to *teach* the common sense method, and afterwards indicate the method by "statement," by going through a few examples. We will work a sum by the rule of common sense.

Ex. If 30 cwt. are carried 15 miles for £5. 8*s*. 9*d*., how far ought 80 cwt. to be carried for £29?

If 30 cwt. for 1305*d*. are carried 15 miles;

$\therefore$ 1 cwt. for 1305*d*. is carried 15×30 miles;

$\therefore$ 1 cwt. for 1*d*. is carried $\frac{15 \times 30}{1305}$ miles;

$\therefore$ 80 cwt. for 1*d*. are carried $\frac{15 \times 30}{1305 \times 80}$ miles;

$\therefore$ 80 cwt. for 6960*d*. are carried $\frac{15 \times 30 \times 6960}{1305 \times 80}$ miles;

$$\therefore \text{Ans.} = \frac{\overset{3}{\cancel{15}} \times \overset{3}{\cancel{30}} \times \overset{\overset{30}{\cancel{240}}}{\cancel{6960}}}{\underset{\underset{\underset{\cancel{29}}{\cancel{87}}}{\cancel{261}}}{\cancel{1305}} \times \underset{\cancel{8}}{\cancel{80}}} \text{ miles} = 30 \text{ miles.}$$

The rules to be observed are:

1. Write down what you are told.

2. Write it down in such a manner that that comes last which is the same kind of thing as the answer.

In the statement method, the rules are:

1. The third term must be the same kind of thing as the answer.

2. The other two terms (which are compared) must be of a like sort.

3. If the answer is greater than the third term, put the greater of the two like things in the second place; if less than the third term, put the less of the two like things in the second place.

4. Multiply the second and third terms together, and divide by the first. This will give the answer.

E.g. If 5 yds. of cloth cost 12*s.*, what would 15 yds. cost?

$$\begin{array}{ccccccc} \text{yds.} & & \text{yds.} & & s. & & s. \\ 5 & : & 15 & :: & 12 & : & \text{Ans.} \end{array}$$

$$\therefore \text{ Ans} = \frac{\overset{3}{\cancel{15}} \times 12}{\cancel{5}} s. = 36s. = £1.\ 16s.$$

This is the right statement. The following, though it will bring out a right answer, has no meaning, and should never be allowed.

$$\begin{array}{ccccccc} \text{yds.} & & s. & & \text{yds.} & & s. \\ 5 & : & 12 & :: & 15 & : & \text{Ans.} \end{array}$$

It has no meaning, because no arithmetical comparison can be instituted between 5 yds. and 12*s.*, &c.

We will give an example of double rule of three by the statement method. *No other statement should ever be allowed.*

If 12 persons spend £160 in 4 months, how many will £853. 6*s.* 8*d.* last for 8 months?

$$\left.\begin{array}{ccc} £. & & £. \\ 160 & : & 853\frac{1}{3} \\ \text{mo.} & & \text{mo.} \\ 8 & : & 4 \end{array}\right\} \begin{array}{cccc} & \text{per.} & & \text{per.} \\ :: & 12 & : & \text{Ans.} \end{array}$$

$$\therefore \text{ Ans.} = \frac{12 \times 853\frac{1}{3} \times 4}{160 \times 8} \text{ persons} = \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{12}} \times \overset{16}{\cancel{2560}} \times \cancel{4}}{\cancel{160} \times \underset{\cancel{2}}{\cancel{8}} \times \cancel{3}} \text{ persons}$$

$$= 32 \text{ persons.}$$

Rules reducible to Proportion.

34. All the money rules, called Interest, Discount, Brokerage, Stocks, and Profit and Loss, are only proportion applied to particular subjects, which involve new definitions. Hence, when the definitions and nomenclature are thoroughly understood, these rules can contain no fresh difficulty.

35. *Simple Interest.*

This rule introduces the following words: Interest, Principal and Amount. The Rate of Interest is the interest on £100 for 1 year. Let us now solve two or three questions. See Note C.

Ex. 1. Find the amount on £250. 12*s.* 6*d.* from March 26, 1840, to Oct. 31, 1842, at 3 per cent.

From March 6, 1840, to Oct. 31, 1842, is 2 years, 219 days $= 2\frac{219}{365}$ years $= 2\frac{3}{5}$ years.

$$\pounds 250.\ 12s.\ 6d. = \pounds 250\frac{5}{8} = \pounds\frac{2005}{8}.$$

First, find the interest.

If £100 in 1 year gains £3;

$$\therefore\ £1 \text{ in } 1 \text{ year gains } £\frac{3}{100};$$

$$\therefore\ £\frac{2005}{8} \text{ in } 1 \text{ year gains } £\frac{3 \times 2005}{100 \times 8};$$

$$\therefore\ £\frac{2005}{8} \text{ in } \frac{13}{5} \text{ years gains } £\frac{3 \times \overset{401}{\cancel{2005}} \times 13}{100 \times 8 \times \cancel{5}}$$

$$= £\frac{15639}{800} = £19.\ 10s.\ 11\tfrac{7}{10}d. \text{ Int.}$$

Add £250 12 6 Principal.

Hence we get £270. 3*s.* $5\frac{7}{10}$*d.* Amt. Ans.

N.B. It will be found convenient, generally, if not always, to work in £'s and fractions of £'s as above, and not to reduce to lower denominations.

It is as well to observe that an easier plan of finding simple interest or amount can generally be adopted than the above, viz. by the ordinary rule, which is, to multiply the principal, the no. of years, and the rate per cent. together, and divide by 100. This will give us the interest. The reason of this rule will not require explanation. Thus, in the above example,

$$\text{Int.} = £\frac{250\frac{5}{8} \times 3 \times 2\frac{3}{5}}{100} = £\frac{2005 \times 3 \times 13}{100 \times 8 \times 5},$$
$$= \dots\dots\dots\dots\dots\dots$$

The reason that we have not employed this method is partly because it is of limited application, and partly that it may be thoroughly seen *in all cases* that Interest and the other money rules are not new rules, but only various applications of proportion adapted to new definitions. See Art. 34.

It might be asked, "Why cannot we find the amount at once in the last sum, beginning thus:

If £100 in 1 year amounts to £103, &c.?"

The answer is, that no proportion is true which involves *at the same time* years and amount. This can easily be proved, for it is clear that, though interest increases in the same proportion as the length of years, amount does not, because a part of it (viz. the principal) remains stationary during the time. Thus, £100 at 5 per cent. in 1 year amounts to £105; in 2 years to £110, *not to* £210.

Ex. 2. At what rate will £220. 12*s*. 6*d*. become £240. 4*s*. $8\frac{2}{3}$*d*. in $3\frac{1}{3}$ years?

Here, £240. 4*s*. $8\frac{2}{3}$*d*. is amt. But years and amount cannot exist together in a proportion sum. Subtract therefore the principal from the amount to get the interest. Thus, we have £220. 12*s*. 6*d*. gains £19. 12*s*. $2\frac{2}{3}$*d*. in $3\frac{1}{3}$ years.

$$£220.\ 12s.\ 6d. = £220\frac{5}{8} = £\frac{1765}{8}.$$

$$£19.\ 12s.\ 2\tfrac{2}{3}d. = £19\frac{11}{18} = £\frac{353}{18}.$$

If $£\frac{1765}{8}$ in $\frac{10}{3}$ years gains $£\frac{353}{18}$;

$\therefore$ £1 in $\frac{10}{3}$ years gains $£\frac{353 \times 8}{18 \times 1765}$;

$\therefore$ £1 in 1 year gains $£\frac{353 \times 8 \times 3}{18 \times 1765 \times 10}$;

$$\therefore\ £100 \text{ in } 1 \text{ year gains } £\frac{\cancel{353} \times 8 \times \cancel{3} \times \overset{\overset{5}{\cancel{10}}}{\cancel{100}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{18}} \times \underset{\cancel{353}}{\cancel{1765}} \times \cancel{10}}$$

$$= £\frac{8}{3} = £2\frac{2}{3};$$

$\therefore$ Ans. $= 2\frac{2}{3}$ per cent. (for we have found the interest on £100 for 1 year, i.e. the rate).

Ex. 3. What sum at $4\frac{2}{3}$ per cent. will become £49. 0*s*. $5\frac{1}{4}$*d*. in $5\frac{1}{4}$ years?

Here we have given the amount, but cannot find the interest, as in order to get it we should have to subtract the principal which we have to find from the given amount. Hence, we must seek some other way to get out a true answer. Let us find the amount on £100 for $5\frac{1}{4}$ years at $4\frac{2}{3}$ per cent.

$$\text{Interest will be } £\frac{\cancel{100} \times 5\frac{1}{4} \times 4\frac{2}{3}}{\cancel{100}}$$

$$= £\frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}} = £\frac{49}{2}$$

$$= £24\frac{1}{2};$$

$$\therefore\ \text{Amount} = £124\frac{1}{2}.$$

The question is thus reduced to the following: If £100 amounts to $£124\frac{1}{2}$, what sum will amount to £49. 9*s*. $5\frac{1}{4}d.$? This is a *proportion* sum, as we have got rid of *years*.

$$£49.\ 9s.\ 5\tfrac{1}{4}d. = £49\frac{7}{320} = £\frac{15687}{320}.$$

If $£124\frac{1}{2}$ is the amount on £100;

$$\therefore\ £1 \text{ is the amount on } £\frac{100 \times 2}{249};$$

$$\therefore\ £\frac{15687}{320} \text{ is the amount on } £\frac{\overset{5}{\cancel{100}} \times \cancel{2} \times \overset{63}{\overset{\cancel{5229}}{\cancel{15687}}}}{\underset{\cancel{83}}{\cancel{249}} \times \underset{8}{\underset{\cancel{16}}{\cancel{320}}}}$$

$$= £\frac{315}{8} = £39\frac{3}{8}$$

$$= £39.\ 7s.\ 6d. \quad \text{Ans.}$$

N.B. The practical rule for solving interest questions involving amount is thus found to be: If possible, get rid of amount, by means of the data (i.e. by subtracting interest or principal from amount). If not possible, find the amount on £100 for the given time at the given rate.

36. *Compound Interest.*

Here the process adopted by us all through the latter rules may be employed. But a far shorter method is to take the ordinary rule and work by decimals.

Ex. 1. Find the amount of £95. 16*s.* 8*d.* for 2 years, at $2\frac{1}{2}$ per cent., comp. int.

£.	*s.*	*d.*
95	16	8
		$2\frac{1}{2}$
191	13	4
47	18	4
2·39	11	8
20		
7·91		
12		
11·00		

∴ Amt. for 1st year = £95. 16*s.* 8*d.* + £2. 7*s.* 11*d.* = £98. 4*s.* 7*d.*

£.	*s.*	*d.*
98	4	7
		$2\frac{1}{2}$
196	9	2
49	2	$3\frac{1}{2}$
2·45	11	$5\frac{1}{2}$
20		
9·11		
12		
$1·37\frac{1}{2}$		

$$37\frac{1}{2} \div 100 = \frac{\overset{3}{\cancel{75}}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{3}{8}.$$

∴ Amt. for 2 years = £98. 4*s.* 7*d.* + £2. 9*s.* $1\frac{3}{8}$*d.* = £100. 13*s.* $8\frac{3}{8}$*d.* Ans.

Ex. 2. Find the comp. int. on £225 for 3 years, at $3\frac{3}{4}$ per cent.

$$\begin{array}{r} £225 \\ 3\frac{3}{4} \\ \hline 675 \\ 168{\cdot}75 \\ \hline 843{\cdot}75 \end{array}$$

∴ Amt. for 1st year = £225 + £8·4375 = £233·4375.

$$\begin{array}{r} £233{\cdot}4375 \\ 3\frac{3}{4} \\ \hline 7003125 \\ 175078125 \\ \hline 875{\cdot}390625 \end{array}$$

∴ Amt. for 2 years = £233·4375 + £8·75390625 = £242·19140625.

$$\begin{array}{r} £242{\cdot}19140625 \\ 3\frac{3}{4} \\ \hline 72657421875 \\ 18164355546875 \\ \hline 908{\cdot}2177734375 \end{array}$$

∴ Amt. for 3 years = £242·19140625 + £9·082177734375 = £251·273583984375.

∴ Ans. = £251·273583984375 − £225 = £26·273583984375 = £26. 5*s*. $5\frac{169}{256}$*d*.

This method might have been adopted in Ex. 1. We should have to begin thus:

$$£95.\ 16s.\ 8d. = £95\frac{5}{6} = £95{\cdot}8\dot{3} = £95{\cdot}83333\ldots$$

Corrections on account of the recurrence would have to be introduced at the different steps.

37. *Discount.*

This rule introduces the new words, discount, bill, present value.

Suppose a man borrows £500, with promise to pay £550 in 3 years time. The £550 is called the bill, and is the amount that is due to the money-lender at the end of 3 years. The £500 is called (very naturally) the present value, and the difference (viz. £50), the discount.

Supposing we want to find the present value of £550, due at the end of 3 years at 5 per cent., the question resolves itself into the old interest question, what principal in 3 years at 5 per cent. will become £550. For the money-lender might be supposed to say to the borrower, "It makes no difference to me whether you pay me £550 in 3 years (as we agreed), or whether you pay me *now* such a sum of money that if I put it into the bank at 5 per cent., I shall find at the end of 3 years that it has become £550." Hence a "bill" is only another name for amount, present value for principal, and discount for interest.

Ex. Find the discount on a bill of £419. 12*s.* 1*d.*, drawn March 6, at 7 months, discounted Sept. 15, at 5 per cent.

Here the bill was drawn March 6, at 7 months, and was therefore due Oct. 9 (allowing for the three days of grace). Now it was discounted Sept. 15, i.e. 24 days before it was due. Hence we have the question:

What principal in 24 days will become £419. 12*s.* 1*d.*, at 5 per cent.?

$$£419.\ 12s.\ 1d. = £419\frac{29}{48} = £\frac{20141}{48}.$$

$$\text{Int. on } £100 \text{ at } 5 \text{ per cent. for } 24 \text{ days} = £\frac{24}{\cancel{365}_{73}} \text{ of } \frac{\cancel{5}}{1}$$

$$= £\frac{24}{73};$$

$$\therefore \text{Amt.} = £100\frac{24}{73} = £\frac{7324}{73}.$$

If $£\frac{7324}{73}$ is amount on £100;

$$\therefore £1 \text{ is amount on } £\frac{100 \times 73}{7324};$$

$$\therefore £\frac{20141}{48} \text{ is amount on } £\frac{\overset{25}{\cancel{100}} \times 73 \times \overset{11}{\cancel{20141}}}{\underset{\cancel{1831}}{\cancel{7324}} \times 48}$$

$$= £\frac{20075}{48} = £418\frac{11}{48}$$

$$= £418.\ 4s.\ 7d.$$

$\therefore$ Discount $= £419.\ 12s.\ 1d. - £418.\ 4s.\ 7d. = £1.\ 7s.\ 6d.$ Ans.

38. *Stocks.*

This rule introduces the words, capital, stock, income, and such new expressions as, "the 5 per cents. are at 90." This expression means that £5 is the interest on £100 stock, which is worth £90.

All questions in stocks proper may be reduced to three or four primitive questions. We will solve a few; in each case supposing that the 5 per cents. are at 90.

Ex. 1. What is the value of £1000 stock?

If £100 stock is worth £90;

$$\therefore \text{£1 stock is worth } £\frac{90}{100};$$

$$\therefore \text{£1000 stock is worth } £\frac{90 \times 10\cancel{0}\cancel{0}}{\cancel{1}\cancel{0}\cancel{0}}$$

$$= £900. \quad \text{Ans.}$$

Ex. 2. How much stock can be obtained for £1000?

If £90 will buy £100 stock;

$$\therefore \text{£1 will buy } £\frac{100}{90} \text{ stock};$$

$$\therefore \text{£1000 will buy } £\frac{100 \times 100\cancel{0}}{9\cancel{0}} \text{ stock}$$

$$= £1111\frac{1}{9} \text{ stock.} \quad \text{Ans.}$$

Ex. 3. What is my income from £1000 stock?

If £100 stock produces £5;

$$\therefore \text{£1 stock produces } £\frac{5}{100};$$

$$\therefore \text{£1000 stock produces } £\frac{5 \times 10\cancel{0}\cancel{0}}{\cancel{1}\cancel{0}\cancel{0}}$$

$$= £50. \quad \text{Ans.}$$

Ex. 4. What is my income from investing £1000?

If £90 produces £5;

$$\therefore \text{£1 produces } £\frac{5}{90};$$

$$\therefore \text{£1000 produces } £\frac{5 \times 100\cancel{0}}{9\cancel{0}}$$

$$= £55\frac{5}{9}$$

$$= £55.\ 11s.\ 1\tfrac{1}{3}d. \quad \text{Ans.}$$

This last is done in the shortest, but not the most natural way. Properly speaking, we ought first to have found the amount of stock purchased, as in Ex. 2. This is £$1111\frac{1}{9}$ stock. Then proceed in the same manner as in Ex. 3, thus:

If £100 stock produces £5;

$$\therefore\ £1 \text{ stock produces } £\frac{5}{100};$$

$$\therefore\ £\frac{10000}{9} \text{ stock produces } £\frac{5 \times 100\cancel{00}}{\cancel{100} \times 9}$$

$$= £55.\ 11s.\ 1\tfrac{1}{3}d. \quad \text{Ans.}$$

The best method of doing examples in stocks is to separate each question into the events that, as a matter of fact, really happened, and consider these events consecutively, e.g.

Ex. 5. A person invests £18150 in the 3 per cents. at $90\frac{3}{4}$, and, on their rising to 91, transfers to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$; what increase does he make thereby in his annual income?

First Question. He buys stock with £18150; find the amount of stock bought.

If £$90\frac{3}{4}$ buys £100 stock;

$$\therefore\ £1 \text{ buys } £\frac{100 \times 4}{363} \text{ stock};$$

$$\therefore\ \pounds18150 \text{ buys } \pounds\frac{100\times4\times\overset{\overset{\overset{50}{\cancel{550}}}{\cancel{6050}}}{\cancel{18150}}}{\underset{\underset{\cancel{11}}{\cancel{121}}}{\cancel{363}}} \text{ stock}$$

$$= \pounds20000 \text{ stock.}$$

Second Question. This £20000 stock he sells, when the stock is at 91. Find how much money he gets for his stock.

If £100 stock is sold for £91;

$$\therefore\ \pounds1 \text{ stock is sold for } \pounds\frac{91}{100};$$

$$\therefore\ \pounds20000 \text{ stock is sold for } \pounds\frac{91\times200\cancel{00}}{\cancel{100}}$$

$$= \pounds18200.$$

Third Question. With this £18200, he buys some more stock, at $97\frac{1}{2}$. Find the amount of stock bought.

If $\pounds97\frac{1}{2}$ buys £100 stock;

$$\therefore\ \pounds1 \text{ buys } \pounds\frac{100\times2}{195} \text{ stock};$$

$$\therefore\ \pounds18200 \text{ buys } \pounds\frac{100\times2\times\overset{\overset{280}{\cancel{3640}}}{\cancel{18200}}}{\underset{\underset{3}{\cancel{39}}}{\cancel{195}}} \text{ stock}$$

$$= \pounds\frac{56000}{3} \text{ stock}$$

$$= \pounds18666\frac{2}{3} \text{ stock.}$$

Since we are asked to find the difference in his income caused by these transactions, we must find his *first* income, when he had £20000 stock in the three per cents., and his *second* income, when he had £$18666\frac{2}{3}$ stock in the $3\frac{1}{2}$ per cents. Hence,

Fourth Question. If £100 stock produces £3;

$$\therefore \text{£}1 \text{ stock produces } \text{£}\frac{3}{100};$$

$$\therefore \text{£}20000 \text{ stock produces } \text{£}\frac{3 \times 200\cancel{00}}{\cancel{1}\cancel{0}\cancel{0}}$$
$$= \text{£}600.$$

Fifth Question. If £100 stock produces £$\frac{7}{2}$;

$$\therefore \text{£}1 \text{ stock produces } \text{£}\frac{7}{2 \times 100};$$

$$\therefore \text{£}\frac{56000}{3} \text{ stock produces } \text{£}\frac{7 \times \overset{280}{\cancel{5}\cancel{6}\cancel{0}\cancel{0}\cancel{0}}}{\cancel{2} \times \cancel{1}\cancel{0}\cancel{0} \times 3}$$
$$= \text{£}\frac{1960}{3}$$
$$= \text{£}653.\ 6s.\ 8d.$$

Sixth Question. Hence the difference in income = £653. 6*s.* 8*d.* – £600

= £53. 6*s.* 8*d.* Ans.

Sometimes, when no ambiguity can arise, the word stock is not expressed. Thus, if it is said that a man bought £1000, sold £1000, transferred £1000 from the 4 per cents. at 90 to the 5 per cents. at 80, or had £1000 in the 5 per cents. at 90, in all these cases the £1000 is clearly stock. If it is said that a man laid out £1000, or invested £1000 in

the 4 per cents. at 90, or that a sum in the 4 per cents. at 90 was worth £1000; in all these cases the £1000 is clearly capital.

39. *Profit and Loss.*

The words here used are cost price, selling price, gain (or loss). It must be remembered that a man always *gains or loses on the cost price.* Thus a loss of 5 per cent. means a loss of £5 on a thing *costing* £100, so that the selling price would be £95.

Ex. 1. If tea be bought at 6*s.* 3*d.* per lb., and sold at 5*s.* 7*d.*, what is the loss per cent.?

Here loss on the cost price 6*s.* 3*d.* is 8*d.*

If on 75*d.* the loss is 8*d.*

$$\therefore \text{ on } 1d. \text{ the loss is } \frac{8}{75}d.;$$

$$\therefore \text{ on } 100d. \text{ the loss is } \frac{8 \times \overset{4}{\cancel{100}}}{\underset{3}{\cancel{75}}}d.$$

$$= \frac{32}{3}d. = 10\frac{2}{3}d.;$$

$$\therefore \text{ Ans. is } 10\frac{2}{3} \text{ per cent.}$$

Obs. Since we are comparing 6*s.* 3*d.* and 100, they must be of the same kind (pence suppose), and since by gain per cent. we mean so much on 100 of the same denomination, it follows that, our answer being $10\frac{2}{3}$ *pence* on 100 *pence*, we get *at once* the result $10\frac{2}{3}$ per cent., *whatever be the denomination.*

Ex. 2. If $5\frac{1}{2}$ per cent. be gained by selling butter at £5. 5*s*. 6*d*. per cwt., what will be the gain per cent., by selling it at 1*s*. 3*d*. per lb.?

As we gain and lose on the *cost price*, it will be necessary, from the first part of the question, to find the cost price, in order to solve the latter part of the same. First then, we will find the cost price.

$$£5.\ 5s.\ 6d. = £5\frac{11}{40} = £\frac{211}{40}.$$

If a thing which is sold for $£105\frac{1}{2}$ costs £100;

$\therefore$ a thing which is sold for £1 costs $£\frac{100 \times 2}{211}$;

$\therefore$ a thing which is sold for $£\frac{211}{40}$ costs $£\frac{\overset{5}{\cancel{100}} \times \cancel{2} \times \cancel{211}}{\cancel{211} \times \underset{\cancel{2}}{\cancel{40}}}$

$$= £5.$$

If 1 lb. is sold for 1*s*. 3*d*.;

$\therefore$ 1 cwt. is sold for $\frac{5}{\cancel{4}} \times \frac{\overset{28}{\cancel{112}}}{1}s. = 140s. = £7$;

$\therefore$ gain on the cwt. is $£7 - £5 = £2$.

If on £5 the gain is £2;

$\therefore$ on £1 the gain is $£\frac{2}{5}$;

$\therefore$ on £100 the gain is $£\frac{2 \times \overset{20}{\cancel{100}}}{\cancel{5}}$

$$= £40;$$

$\therefore$ Ans. = 40 per cent.

The above may be done a shorter way, but it is unsatisfactory, as the ordinary student would probably find it difficult to explain the reason of his method. It is well to solve all questions in Profit and Loss by the single simple principle that all gain or loss is on the cost price. No method but the above should be allowed. All other methods are dangerous.

Ex. 3. If a person purchase pins when they are 18 in a row, and sell them 11 in a row at the same price, how much is his gain per cent.?

As a person gains and loses on the *cost price*, we must imagine a cost price, to work the question simply and clearly.

Suppose 18 pins to cost 18 pence;

$\therefore$ 11 pins are sold for 18 pence.

But since 18 pins cost 18 pence;

$\therefore$ 11 pins cost 11 pence;

$\therefore$ 11 pins cost 11 pence and are sold for 18 pence;

$\therefore$ 7*d.* is the gain on the cost price 11*d.*

If on 11*d.* the gain is 7*d.*;

$$\therefore \text{ on } 1d. \text{ the gain is } \frac{7}{11}d.$$

$$\therefore \text{ on } 100d. \text{ the gain is } \frac{7 \times 100}{11}d.$$

$$= 63\frac{7}{11}d.$$

$$\therefore \text{ Ans.} = 63\frac{7}{11} \text{ per cent.}$$

40. *Proportional parts.*

This rule really only includes an easy class of questions involving knowledge of fractions and proportion, and there is no reason why it should usurp a high place in Arithmetic.

If a sum of money is to be divided among A and B in the ratio of $3 : 5$, it is clear that as often as A has £3, B will have £5. Therefore, out of every £8 that is divided between them, A has £3 and B has £5. Therefore A will have $\frac{3}{8}$ of the whole, B $\frac{5}{8}$ of the whole.

We will solve two questions.

Ex. 1. A and B engaged in trade, their capitals being in the ratio of $4 : 5$; and, at the end of 3 months, they withdrew respectively $\frac{2}{3}$ and $\frac{3}{4}$ of their capitals: how should they divide their whole gain, £335, at the end of the year?

Suppose A to have £4;

$\therefore$ B will have £5.

Since A withdraws $\frac{2}{3}$ of his capital, he will have $\frac{1}{3}$ of £4 $\left(=£\frac{4}{3}\right)$ remaining.

Since B withdraws $\frac{3}{4}$ of his capital, he will have $\frac{1}{4}$ of £5 $\left(=£\frac{5}{4}\right)$ remaining.

Now a capital of £4 for 3 months is equivalent to £12 for 1 month.

And a capital of £$\frac{4}{3}$ for 9 months is equivalent to £12 for 1 month;

∴ (adding) we may represent *A*'s capital at £24 for 1 month.

Again, a capital of £5 for 3 months is equivalent to £15 for 1 month.

And a capital of £$\frac{5}{4}$ for 9 months is equivalent to £$11\frac{1}{4}$ for 1 month;

∴ (adding) we may represent *B*'s capital at £$26\frac{1}{4}$ for 1 month.

Hence, if we divide the profits, £335, in the ratio of the two capitals, £24 and £$26\frac{1}{4}$, we shall get the proper answer.

(Observe, a capital of £4 for 3 months is equivalent to a capital of £12 for 1 month, for by an *equivalent* capital we mean a capital that will produce the same profits. Of course if £4 were the *profits* in 3 months, the profits in 1 month would be £$\frac{4}{3}$.)

Ex. 2. If 8 oz. of gold, 10 carats fine, and 2 oz., 11 carats fine, be mixed with 6 oz. of unknown fineness, and that of the mixture be 12 carats, what was the unknown fineness?

By 8 oz. of gold, 10 carats fine, we mean that $\frac{10}{24}$ of the 8 oz. is gold, the rest being alloy, see Art. 4. In this and all similar questions, reduce to, and work in ordinary Tr. weight, and if the answer is to be given in carats, re-

duce to 24ths of the whole mass in question. The number of 24ths will give us the fineness required.

8 oz. of gold, 10 carats fine, contain $\frac{80}{24}$ oz. of pure gold.

2 oz. of gold, 11 carats fine, contain $\frac{22}{24}$ oz. of pure gold.

$\therefore$ (adding) 10 oz. of the mixed gold contain $\frac{102}{24}$ oz. of pure gold.

But the whole 16 oz. of the mixed gold, 12 carats fine, contain $\frac{192}{24}$ oz. of pure gold;

$\therefore$ the 6 oz. of gold introduced contain $\frac{90}{24}$ oz. of pure gold;

$\therefore$ 1 oz. of gold introduced contains $\frac{15}{24}$ oz. of pure gold;

$\therefore$ the unknown fineness is 15 carats. Ans.

(For in every oz. of the new gold we have proved that 15 parts out of 24 are pure gold.)

41. If *A* can do $\frac{3}{5}$ of a piece of work in 7 days, $\frac{1}{2}$ of which *A* and *B* together can do in 5, how long would *B* alone be doing $\frac{5}{7}$ of it?

Since *A* can do $\frac{3}{5}$ of the work in 7 days;

$\therefore$ he can do the whole work in $\frac{5}{3}$ of $7 = \frac{35}{3}$ days;

$\therefore$ he can do $\frac{3}{35}$ of the work in 1 day.

Since A and B can do $\frac{1}{2}$ of the work in 5 days;

$\therefore$ they can do the whole work in 10 days;

$\therefore$ they can do $\frac{1}{10}$ of the work in 1 day;

$\therefore$ B alone can do $\frac{1}{10} - \frac{3}{35}$ of the work in 1 day;

that is, $\frac{1}{70}$ of the work in 1 day.

If B can do $\frac{1}{70}$ of the work in 1 day;

$\therefore$ he can do the whole work in 70 days;

$\therefore$ he can do $\frac{5}{7}$ of the work in $\frac{5}{7}$ of 70 days;

that is, in 50 days. Ans.

42. If two fruit women were willing to exchange 12 apples for 49 pears, 121 nuts for 25 pears, 6 oranges for 5 plums, 154 nuts for 45 oranges; how many apples would they be willing to exchange for 77 plums?

$$\begin{aligned} 12 \text{ apples} &= 49 \text{ pears,} \\ 25 \text{ pears} &= 121 \text{ nuts,} \\ 154 \text{ nuts} &= 45 \text{ oranges,} \\ 6 \text{ oranges} &= 5 \text{ plums,} \\ 77 \text{ plums} &= x \text{ apples.} \end{aligned}$$

$$\therefore \text{Ans.} = \frac{\overset{4}{\cancel{12}} \times \overset{\cancel{5}}{\cancel{25}} \times \overset{\overset{2}{\cancel{14}}}{\cancel{154}} \times \overset{2}{\cancel{6}} \times \overset{\cancel{11}}{\cancel{77}}}{\underset{\cancel{7}}{\cancel{49}} \times \underset{\cancel{11}}{\cancel{121}} \times \underset{\underset{3}{\cancel{9}}}{\cancel{45}} \times \cancel{5}} \text{ apples}$$

$= 16$ apples. Ans.

We shall not explain the reason of the above. The student may easily verify it by a series of proportions. It is called the "chain rule," and is useful in working questions of "Exchange" in a ready and simple manner. It will be observed in the two upright columns *that each of them contains all the different denominations.*

43. *Per centage.*

A large number of the latter examples, after proportion, may be worked out in a different manner by a careful consideration of the words "per centage."

Thus, 2 per cent. means that 2 articles out of 100 of the same articles are taken into consideration; that is, we are considering $\frac{2}{100}$ of the whole.

If a man loses 5 per cent. of his money, in any way, it means that he loses $\frac{5}{100}$ of his money. He will have remaining $\frac{95}{100}$. We can thus get rid of proportion, in examples of per centage, and reduce per centage to fractions.

On the whole, however, we object to this method, because ambiguities are likely to arise from the looseness with which the expression "so much per cent." is used in common language. To make questions of this sort free from ambiguity, we should either have to define scientifically popular language, or explain carefully the exact sense in which we intend to use the words in the different cases.

44. We will finish with certain practical rules which will be found *very useful* for checking errors, and in many

ways stimulating the ease and rapidity of correct work. We do not propose to prove them generally, or explain on what principles they depend. Some may be proved by simple Arithmetic, others by Arithmetic diluted with Algebra; while the proof of others belongs entirely to the region of Algebra. But the results are important and easy to be remembered.

a. A number is divisible by 2, 4, 8, respectively, if the number represented by its last 1, 2, 3, figures respectively is divisible by 2, 4, 8, respectively.

The rule for 4 can be proved thus:

100 is divisible by 4, therefore any number of hundreds is divisible by 4; if then a number is divisible by 4, the remainder found by removing the hundreds, that is, the number represented by the last two digits, must be divisible by 4. So for 8, 16, 32 ...

A number is divisible by 5, if it ends in a 5 or a cypher.

A number is divisible by 10, if it ends in a cypher.

A number is divisible by 3, 9, respectively, if the sum of its digits is divisible by 3, 9, respectively. See Note D.

A number is divisible by 11, if the difference of the sums of its digits in the odd and even places is divisible by 11. See Note D.

The rule for 7 is more complex than ordinary division by 7.

The rule for 6 is the combination of the rules for 2 and 3; for 12, the combination of the rules for 3 and 4; &c. Care must be taken to split 12 into its factors 4, 3 (not 6, 2 for instance); 24 into its factors 8, 3; 75 into its factors 25, 3, &c.

N.B. These rules are useful, in reducing fractions to their lowest terms, in cancelling, L.C.M., G.C.M., &c.

β. The shortest way of multiplying a number by 25 is, to add two cyphers and divide by 4.

The shortest way of multiplying a number by 125 is, to add three cyphers and divide by 8.

So, the shortest way of dividing a number by 25 is, to mark off two decimal places, and multiply by 4.

And the shortest way of dividing a number by 125 is, to mark off three decimal places, and multiply by 8.

N.B. It may be observed that an odd number is divisible by 25 if it ends in 25 or 75; by 125 if it ends in 125, 625, 375, 875.

These rules are useful in reducing fractions to decimals, and conversely; generally in cases where the decimals terminate. They are also useful in other work.

γ. Let us take a fraction *in its lowest terms*, which we wish to reduce to a decimal. Then the following rules hold.

1. If the denominator of the fraction consists *only* of powers of 2 and 5 as factors, the resulting decimal will terminate. Art 27.

2. If the denominator of the fraction has no factor of 2 or 5, the resulting decimal will be a pure circulator. Art. 27.

3. If the denominator of the fraction consists partly of powers of 2 or 5, or both, as factors, and partly of other factors, the resulting decimal will be a mixed circulator. Art. 27.

4. The number of figures in the terminating decimal, in (1), will be that power of 2 or 5 in the denominator of the fraction which happens to be the greater. Art. 27.

5. The number of figures in the period of the decimal, in (2), will be a measure of some measure diminished by unity of the denominator of the fraction.

6. The number of figures *before* the period of the decimal, in (3), will follow rule (4); *in* the period of the decimal, will follow rule (5). Art. 27.

7. When the decimal is a pure circulator, and the period a full period, the same period will occur, only commencing at a different figure of the period, for every fraction with the same denominator, but a different numerator. See Art. 32.

8. When the decimal is a pure circulator, and the period a full period, every figure in the last half of the period will be a complement to 9 of every corresponding figure in the first half of the period.

[N.B. By a full period, we mean a period that is the largest possible, i.e., one less than the denominator of the fraction. See Rule (5)].

We will exemplify rules (7) and (8). We shall find that

$$\frac{1}{19} = \cdot\dot{0}5263157894736842\dot{1}.$$

Here (2) and (5) are satisfied, and the number of figures in the period being $19 - 1$, it is a full period. Hence we can find $\frac{2}{19}$, $\frac{3}{19}$, &c., at once. Thus:

$$\frac{2}{19} = \cdot\dot{1}0526315789473684\dot{2},$$

$$\frac{3}{19} = \cdot\dot{1}5789473684210526\dot{3},$$

$$\frac{15}{19} = \cdot\dot{7}8947368421052631\dot{5}.$$

We began this last $\cdot\dot{7}89\ldots$ because there were only three larger fractions than $\frac{15}{19}$, viz. $\frac{16}{19}$, $\frac{17}{19}$, $\frac{18}{19}$; and there were only 3 decimals beginning with higher figures than $\cdot\dot{7}89\ldots$, viz. $\cdot\dot{8}42\ldots$, $\cdot\dot{8}94\ldots$ and $\cdot\dot{9}47\ldots$ So for any other numerator. In Art. 32, we have already discussed the important fractions $\frac{1}{7}$, $\frac{2}{7}$, &c., in like manner.

Again, take one of the above fractions, as $\frac{15}{19}$, and divide the period by a mark into its two halves, thus:

$$\frac{15}{19} = \cdot\dot{7}89473684{,}21052631\dot{5}.$$

Hence (8) is clearly seen to be true, for $7+2=9$, $8+1=9$, $9+0=9$, $4+5=9$, &c. So for any other numerator, or for any of the fractions $\frac{1}{7}$, $\frac{2}{7}$, &c., in Art. 32.

Ex. The first 29 figures of the period, for the fraction $\frac{1}{59}$, are found to be

$$\cdot\dot{0}1694915254237288135593220338\ldots\ldots$$

Complete the period.

There are other very curious properties about the remainders, in reducing fractions to decimals, and about the periods, both when full and incomplete, but we do not give them here, partly because they are not so important as the above, partly because it is not easy to express them sufficiently generally or concisely for practical work.

δ. 6*s*. 8*d*. is $\frac{1}{3}$ of a £; 2*s*. 6*d*. is $\frac{1}{8}$ of a £.

If we wish to reduce shillings, pence, and farthings to the fraction of a £, the denominator of the resulting fraction *can have no other factors* than 3, 5, and powers of 2.

NOTES.

NOTE A. *Multiplication of fractions.*

By "multiplication" we mean the repetition of a number a certain number of times. Thus, 7×4 means 7 repeated 4 times, or $7+7+7+7$, or 28. So 7 apples multiplied by 4 is 28 apples. From this definition it is clear that the multiplier must be abstract and a whole number. According to our definition then, it is clear that we could not multiply by any of the following quantities, 4 apples, $\frac{4}{5}$, $+4$, -4. Of course, on the other hand, we could multiply any of them, 4 apples, $\frac{4}{5}$, $+4$, -4, by any abstract integer. It is no matter what the multiplicand may be, it is only necessary that the multiplier should be of a certain character.

If then we are to talk for the future of multiplying by such things as 4 apples, $\frac{4}{5}$, $+4$, -4, it can only be by altering the original meaning of the word multiplication, or at least extending and generalising it. This is done repeatedly in algebra, and thus we get the well-known results,

$$+a \times +b = +ab,$$
$$+a \times -b = -ab,$$
$$-a \times +b = -ab,$$
$$-a \times -b = +ab.$$

And again,

$$a^m \times a^n = a^{m+n}, \ a^m \div a^n = a^{m-n}, \ (a^m)^n = a^{mn},$$

whether m *and* n *be positive, negative, fractional, integral, or zero.*

If again, in Arithmetic, we allow ourselves to use the expression, 4 ft. × 5 ft. = 20 sq. ft., (as in Art. 5) it can either be by supposing that expression to be the abbreviation for the proposition "if we multiply the number of feet in the length of any rectangle by the number of feet in its breadth, we shall get the number of square feet in its area," as is fully explained in the same Article, or we must suppose the meaning of multiplication to be defined anew to suit the case, something for instance as follows.

"A square foot is said to be obtained geometrically by multiplying the foot of its length by the foot of its breadth." Hence it wil be seen (as in Art. 5) that the area of any rectangle would be obtained by multiplying its length by its breadth.

Again, in fractions, we do use such expressions as $\frac{4}{7} \times \frac{5}{9}$, $4 \times \frac{5}{7}$, &c. This is called multiplication of fractions. Of course then here also we must redefine multiplication. (N.B. It is clear it will not be necessary to do so for such cases as $\frac{5}{7} \times 4$.)

The new definition may be given in various ways. We will here state and discuss a definition.

Def. We are said to multiply two fractions together if we multiply the numerators for a new numerator, and the denominators for a new denominator.

This seems at first sight a very arbitrary definition. Any one has a right to ask, Whence did such a definition arise? To shew this, let us take a case, e.g. $\frac{3}{7} \times \frac{4}{5}$.

$\frac{3}{7} \times 4 = \frac{12}{7}$. But we have to multiply not by 4, but by $\frac{4}{5}$, i.e. by $\frac{1}{5}$ of 4. Hence *it will be convenient to agree* that $\frac{3}{7} \times \frac{4}{5}$, or $\frac{3}{7} \times \frac{1}{5}$ of 4 shall mean $\frac{1}{5}$ of $\left(\frac{3}{7} \times 4\right)$, i.e. $\frac{1}{5}$ of $\frac{12}{7}$; which can be easily proved to be $\frac{12}{35}$ (Art. 13). This gives us the ordinary Arithmetical rule.

The important thing here is not to mistake the above for a proof. *It is no proof at all.* It is a convenient assumption, or convention, or new definition of the word "multiplication." Any other definition might have been taken, but this suits us best. The italicised words must be remembered: "it will be convenient to agree."

It might still be asked, however, why the above assumption is more convenient than any other. This we will proceed to explain.

It is clear that $\frac{3}{7} \times 4 = \frac{12}{7}$. Again, it is clear that $\frac{3}{7} \times \frac{12}{3} = \frac{36}{21}$. For $\frac{12}{3} = 4$, and therefore the case in point is $\frac{3}{7} \times 4$, which equals $\frac{12}{7}$, which equals $\frac{36}{21}$. Since also in the first instance, instead of 4 we may write $\frac{4}{1}$, it is clear that we shall not get a wrong answer in the case of

$\frac{3}{7} \times 4$, if for 4 we write either $\frac{4}{1}$ or $\frac{12}{3}$, and then multiply the numerators for a new numerator, and the denominators for a new denominator. In other words, this rule will apply whenever our multiplier is an integer, even if in the form of a fraction. This shews us why our definition is a convenient one when our multiplier is not an integer (in which case multiplication has lost its original meaning), because we shall thus have one rule for all cases; in other words, the form of the answer will not have to be altered to suit the different cases. For instance, we thus know that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ and have not to enquire what $\frac{c}{d}$ is. Otherwise we should have to say "If $\frac{c}{d}$ is really an integer, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. If, however, $\frac{c}{d}$ is not an integer, $\frac{a}{b} \times \frac{c}{d} = \ldots$" whatever our definition of multiplication of fractions makes it equal.

As it is very important that the student should thoroughly understand that there is no meaning in the words "Prove the rule for the multiplication of fractions," or "Shew that $\frac{3}{7} \times \frac{4}{5} = \frac{12}{35}$," but only in "Illustrate the rule," or "Shew why we say that $\frac{3}{7} \times \frac{4}{5} = \frac{12}{35}$," or "Shew that $\frac{3}{7} \times \frac{4}{5}$ is not necessarily equal to $\frac{12}{35}$;" and as the quasi proof given above is often mistaken for a real proof, we will discuss another instance in the same sort of manner, and come to a conclusion different from that which is usually adopted.

5^7 means the repetition of 5 as a factor 7 times;

$\therefore$ 5^{-7} means the repetition of 5 as a factor -7 times;

$\therefore$ 5^{-7} means the repetition of -5 as a factor 7 times;

$\therefore$ $5^{-7} = (-5)^7 = -78125$.

Now this result is not true. Not true, that is to say, *necessarily*. However unwise, it would not be wrong to give a definition of multiplication in the above case, which should make the above result true. But even if the new definition should give that result, the *proof* would still be false—indeed it is manifest there would be no proof possible; only the application of the new definition.

One more word then with regard to results.

$\frac{3}{7} \times 4$ is necessarily $= \frac{12}{7}$.

$\frac{3}{7} \times \frac{4}{5}$ is equal to $\frac{12}{35}$ by a convention governed by common sense, but perfectly arbitrary.

NOTE B. *Division of fractions.*

We now come to division of fractions. The rule given is—Invert the divisor and multiply. The question is—Can this rule be *proved* or no, or if it can, in what manner?

If we allow ourselves the two definitions of division given in Art. 3, this rule can be *proved;* thereby differing from multiplication. The proof will be as follows.

It is clear that $\frac{3}{5} \div 7 = \frac{3}{35}$ by the 2nd def. of division.

This agrees with the rule, for

$$\frac{3}{5} \div 7 = \frac{3}{5} \div \frac{7}{1} = \frac{3}{5} \times \frac{1}{7} = \frac{3}{35}.$$

Again, it is clear that $\frac{3}{5} \div \frac{7}{5} = \frac{3}{7}$. This is clear from the 1st definition of division. This also agrees with the rule, for

$$\frac{3}{5} \div \frac{7}{5} = \frac{3}{\cancel{5}} \times \frac{\cancel{5}}{7} = \frac{3}{7}.$$

Lastly, let us consider $\frac{3}{5} \div \frac{4}{7}$. This is the same as $\frac{21}{35} \div \frac{20}{35}$. The result of this is $\frac{21}{20}$ by the 1st definition of division. And by the rule also we obtain

$$\frac{3}{5} \div \frac{4}{7} = \frac{3}{5} \times \frac{7}{4} = \frac{21}{20}.$$

Hence, in all cases the rule is proved.

We do not like this proof, partly because objections might be raised against it as not compatible with the only definition of fractions we have at present thought it convenient to give, and more especially because we think it a better plan to treat division of fractions in the same manner as we have already treated multiplication of fractions. We leave it to the ingenuity of the student to do this.

Or we might illustrate the rule in the following manner.

Division reverses multiplication. Since then $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$, it follows that $\frac{12}{35} \div \frac{4}{7} = \frac{3}{5}$. It is clear that this quotient may be obtained by inverting the divisor and multiplying. Hence the rule.

N.B. The whole theory of multiplication and division of fractions may be illustrated and dealt with in a different and interesting manner, by extending our definition of a fraction, and introducing axioms, or by making a separation between pure arithmetic and algebraical arithmetic, and laying down the results in parallel columns. It will however probably be felt that we have already dwelt on this subject quite sufficiently.

NOTE C. Every sum worked out in the text, from Art. 35 inclusive to the end, has been handled by the "common sense" method of proportion. We propose here to indicate the course of proof of the same sums, proceeding by the method of statement instead. We conceive that a student might reasonably be allowed to take his choice of either method to adopt.

Art. 35. Ex. 1. Find the amount on £250. 12*s.* 6*d.* from March 26, 1840, to Oct. 31, 1842, at 3 per cent.

From March 6, 1840, to Oct. 31, 1842, is 2 years, 219 days $= 2\frac{219}{365}$ years $= 2\frac{3}{5}$ years $= \frac{13}{5}$ years.

$$£250.\ 12s.\ 6d. = £250\frac{5}{8} = £\frac{2005}{8}.$$

First, find the interest.

$$\left.\begin{array}{rcl} £. & & £. \\ 100 & : & \frac{2005}{8} \\ \text{year.} & & \text{years.} \\ 1 & : & \frac{13}{5} \end{array}\right\} :: \begin{array}{c} £. \\ 3 \end{array} : \text{Int.}$$

Hence the interest, and hence the amount.

Ex. 2. At what rate will £220. 12*s*. 6*d*. become £240. 4*s*. $8\frac{2}{3}$*d*. in $3\frac{1}{3}$ years?

$$£220.\ 12s.\ 6d. = £220\frac{5}{8} = £\frac{1765}{8}.$$

Int. = £240. 4*s*. $8\frac{2}{3}$*d*. − £220. 12*s*. 6*d*. = £19. 12*s*. $2\frac{2}{3}$*d*. $= £19\frac{11}{18} = £\frac{353}{18}$.

$$\left.\begin{array}{l} \overset{£.}{\frac{1765}{8}} : \overset{£.}{100} \\ \text{years. year.} \\ 3\frac{1}{3} : 1 \end{array}\right\} :: \overset{£.}{\frac{353}{18}} : \text{Ans.}$$

∴ Required rate = ...

Ex. 3. What sum will become £49. 0*s*. $5\frac{1}{4}$*d*. in $5\frac{1}{4}$ years, at $4\frac{2}{3}$ per cent.?

Interest on £100 for $5\frac{1}{4}$ years, at $4\frac{2}{3}$ per cent., will be

$$£\frac{100 \times 5\frac{1}{4} \times 4\frac{2}{3}}{100}$$

$$= £\frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}}$$

$$= £\frac{49}{2}$$

$$= £24\frac{1}{2};$$

$$\therefore \text{ Amount} = £124\frac{1}{2} = £\frac{249}{2}.$$

$$£49.\ 9s.\ 5\tfrac{1}{4}d. = £49\frac{7}{320} = £\frac{15687}{320};$$

$$\therefore \quad \overset{£.}{\frac{249}{2}} : \overset{£.}{\frac{15687}{320}} :: \overset{£.}{100} : \text{Ans.}$$

Hence the required principal.

Art. 36. Exs. 1 and 2. These do not require to be considered here, as they have not been done by proportion in the text.

Art. 37. Find the discount on a bill of £419. 12*s.* 1*d.*, drawn March 6, at 7 months, discounted Sept. 15, at 5 per cent.

Here the bill was drawn March 6, at 7 months, and was therefore due Oct. 9 (allowing for the three days of grace). Now it was discounted Sept. 15, i.e. 24 days before it was due.

$$£419.\ 12s.\ 1d. = £419\frac{29}{48} = £\frac{20141}{48}.$$

$$\text{Interest on £100, at 5 per cent. for 24 days} = £\frac{24}{\underset{73}{\cancel{365}}} \times \frac{\cancel{5}}{1}$$

$$= £\frac{24}{73};$$

$$\therefore \text{ Amt.} = £100\frac{24}{73} = £\frac{7324}{73}.$$

$$\overset{£.}{\frac{7324}{73}} : \overset{£.}{\frac{20141}{48}} :: \overset{£.}{100} : \text{Present value.}$$

Hence the present value, and hence the required discount.

It will be observed that it is not sufficient in the first two terms, to compare money merely with money, nor in the third term, to have it merely the same denomination as the answer. Care must be taken that the first two terms should be *both amounts*, and that the third term should be *present value*, the same as the answer.

N.B. If a bill fall due on a day that has no existence, as the 31st of a month of 30 days, or the 29th, 30th, or 31st of Feb. (in leap year, the 30th or 31st of Feb. only), it is considered to be *nominally* due on the last day of the month in question, and therefore *legally* due on the 3rd of the following month.

Art. 38. Ex. 1. What is the value of £1000 stock, in the 5 per cents. at 90?

$$\begin{array}{ccccccc} \text{£. st.} & & \text{£. st.} & & \text{£.} & & \text{£.} \\ 100 & : & 1000 & :: & 90 & : & \text{Ans.} \end{array}$$

Ex. 2. How much stock can be obtained for £1000, in the 5 per cents. at 90?

$$\begin{array}{ccccccc} \text{£.} & & \text{£.} & & \text{£. st.} & & \text{£. st.} \\ 90 & : & 1000 & :: & 100 & : & \text{Ans.} \end{array}$$

Ex. 3. What is my income from £1000 stock, in the 5 per cents. at 90?

$$\begin{array}{ccccccc} \text{£. st.} & & \text{£. st.} & & \text{£.} & & \text{£.} \\ 100 & : & 1000 & :: & 5 & : & \text{Ans.} \end{array}$$

Ex. 4. What is my income from investing £1000 in the 5 per cents. at 90?

$$\begin{array}{ccccccc} \text{£.} & & \text{£.} & & \text{£.} & & \text{£.} \\ 90 & : & 1000 & :: & 5 & : & \text{Ans.} \end{array}$$

N.B. Here the first two terms are *both capital;* the last two, *income.*

Ex. 5. A person invests £18150 in the 3 per cents. at $90\frac{3}{4}$, and, on their rising to 91, transfers to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$; what increase does he make thereby in his annual income?

$$\overset{\text{£.}}{90\tfrac{3}{4}} : \overset{\text{£.}}{18150} :: \overset{\text{£. st.}}{100} : \text{stock purchased};$$

$$\therefore \text{ Stock purchased} = £\frac{\overset{\overset{\overset{50}{\cancel{550}}}{\cancel{6050}}}{\cancel{18150}} \times 100 \times 4}{\underset{\underset{\cancel{11}}{\cancel{121}}}{\cancel{363}}} \text{ stock}$$

$$= £20000 \text{ stock.}$$

$$\overset{\text{£. st.}}{100} : \overset{\text{£. st.}}{20000} : \overset{\text{£.}}{91} : \text{cash received};$$

$$\therefore \text{ Cash received} = £\frac{91 \times 200\cancel{00}}{\cancel{100}}$$

$$= £18200.$$

$$\overset{\text{£.}}{97\tfrac{1}{2}} : \overset{\text{£.}}{18200} :: \overset{\text{£. st.}}{100} : \text{fresh stock purchased};$$

$$\therefore \text{ fresh stock purchased} = £\frac{100 \times \overset{\overset{280}{\cancel{3640}}}{\cancel{18200}} \times 2}{\underset{\underset{3}{\cancel{39}}}{\cancel{195}}} \text{ stock}$$

$$= £\frac{56000}{3} \text{ stock.}$$

$$\begin{array}{ccccccc} \text{£. st.} & & \text{£. st.} & & \text{£.} & & \\ 100 & : & 20000 & :: & 3 & : & \text{first income;} \end{array}$$

$$\therefore \text{ first income} = £\frac{3 \times 200\cancel{00}}{\cancel{100}}$$

$$= £600.$$

$$\begin{array}{ccccccc} \text{£. st.} & & \text{£. st.} & & \text{£.} & & \\ 100 & : & \frac{56000}{3} & :: & 3\frac{1}{2} & : & \text{second income;} \end{array}$$

$$\therefore \text{ second income} = £\frac{\overset{280}{\cancel{56000}} \times 7}{\cancel{2} \times 3 \times \cancel{100}}$$

$$= £\frac{1960}{3}$$

$$= £653.\ 6s.\ 8d.;$$

$\therefore$ required difference of income = £653. 6*s.* 8*d.* – £600,

= £53. 6*s.* 8*d.* Ans.

A single proportion might have been made to replace the third and fifth of the above proportions, as thus:

$$\begin{array}{ccccccc} \text{£.} & & \text{£.} & & \text{£.} & & \\ 97\frac{1}{2} & : & 18200 & :: & 3\frac{1}{2} & : & \text{second income.} \end{array}$$

We prefer the method adopted notwithstanding, as it exhibits the natural progression of the events, and puts the whole working in a clearer and more logical light.

Art. 39. Ex. 1. If tea be bought at 6*s.* 3*d.* per lb., and sold at 5*s.* 7*d.*, what is the loss per cent.?

Here loss on the cost price 6*s.* 3*d.* is 8*d.*

$$\begin{array}{ccccccc} d. & & d. & & d. & & d. \\ 75 & : & 100 & :: & 8 & : & \text{Ans.} \end{array}$$

N.B. Here the first two are cost price, the last two are loss.

Ex. 2. If $5\frac{1}{2}$ per cent. be gained by selling butter at £5. 5*s*. 6*d*. per cwt., what will be the gain per cent., by selling it at 1*s*. 3*d*. per lb.?

$$£5.\ 5s.\ 6d. = £5\frac{11}{40} = £\frac{211}{40}.$$

$$\overset{£.}{105\frac{1}{2}} : \overset{£.}{\frac{211}{40}} :: \overset{£.}{100} : \text{cost price};$$

$$\therefore \text{ cost price} = £\frac{\cancel{211} \times \overset{5}{\cancel{100}} \times \cancel{2}}{\underset{\cancel{2}}{\cancel{40}} \times \cancel{211}}$$

$$= £5.$$

$$\left(£105\frac{1}{2} \text{ and } £\frac{211}{40} \text{ are both selling prices.}\right)$$

If 1 lb. is sold for 1*s*. 3*d*.;

$$\therefore 1 \text{ cwt. is sold for } \frac{5}{\cancel{4}} \times \frac{\overset{28}{\cancel{112}}}{1} s. = 140s. = £7;$$

$$\therefore \text{ gain on 1 cwt.} = £7 - £5 = £2.$$

$$\overset{£.}{5} : \overset{£.}{100} :: \overset{£.}{2} : \text{gain per cent.}$$

$$\therefore \text{ Ans.} = \frac{2 \times \overset{20}{\cancel{100}}}{\cancel{5}} \text{ per cent.}$$

$$= 40 \text{ per cent.}$$

N.B. Here £5 and £100 are both *cost prices*.

Ex. 3. If a person purchase pins when they are 18 in a row, and sell them 11 in a row at the same price, how much is his gain per cent.?

Suppose 18 pins to cost 18 pence;

$\therefore$ 11 pins are sold for 18 pence.

But since 18 pins cost 18 pence;

$\therefore$ 11 pins cost 11 pence;

$\therefore$ 11 pins cost 11 pence, and are sold for 18 pence;

$\therefore$ 7 pence is the gain on the cost price 11 pence.

$$\overset{d.}{11} : \overset{d.}{100} :: \overset{d.}{7} : \text{Ans.}$$

N.B. 11*d.* and 100*d.* are both cost prices; 7*d.* and the answer are both gains.

Art. 42. If two fruit-women were willing to exchange 12 apples for 49 pears, 121 nuts for 25 pears, 6 oranges for 5 plums, 154 nuts for 45 oranges; how many apples would they be willing to exchange for 77 plums?

$$\overset{\text{pl.}}{5} : \overset{\text{pl.}}{77} :: \overset{\text{or.}}{6} : \text{no. of oranges};$$

$$\therefore\ 77 \text{ plums are equivalent to } \frac{6 \times 77}{5} \text{ oranges.}$$

$$\overset{\text{or.}}{45} : \overset{\text{or.}}{\frac{6 \times 77}{5}} :: \overset{\text{nuts.}}{154} : \text{no. of nuts.}$$

$$\therefore\ 77 \text{ plums} \left(= \frac{6 \times 77}{5} \text{ oranges}\right) \text{ are equivalent to}$$

$$\frac{6 \times 77 \times 154}{5 \times 54} \text{ nuts.}$$

$$\begin{array}{ccccccc} \text{nuts.} & & \text{nuts.} & & \text{pears.} & & \\ 121 & : & \dfrac{6 \times 77 \times 154}{5 \times 45} & :: & 25 & : & \text{no. of pears;} \end{array}$$

$$\therefore\ 77 \text{ plums} \left(= \frac{6 \times 77 \times 154}{5 \times 45} \text{ nuts}\right) \text{ are equivalent to}$$

$$\frac{6 \times 77 \times 154 \times 25}{5 \times 45 \times 121} \text{ pears.}$$

$$\begin{array}{ccccccc} \text{pears.} & & \text{pears.} & & \text{app.} & & \\ 49 & : & \dfrac{6 \times 77 \times 154 \times 25}{5 \times 45 \times 121} & :: & 12 & : & \text{no. of apples.} \end{array}$$

$$\therefore\ 77 \text{ plums} \left(= \frac{6 \times 77 \times 154 \times 25}{5 \times 45 \times 121} \text{ pears}\right) \text{ are equivalent to}$$

$$\frac{\overset{2}{\cancel{6}} \times \overset{\cancel{11}}{\cancel{77}} \times \overset{\overset{2}{\cancel{14}}}{\cancel{154}} \times \overset{\cancel{5}}{\cancel{25}} \times \overset{4}{\cancel{12}}}{\cancel{5} \times \underset{\underset{3}{\cancel{9}}}{\cancel{45}} \times \underset{\cancel{11}}{\cancel{121}} \times \underset{\cancel{7}}{\cancel{49}}} \text{ apples}$$

$$= 16 \text{ apples.} \quad \text{Ans.}$$

It would have been shorter to have cancelled at the different stages of the work, but we had the further object in view of explaining the principle of the "chain rule."

NOTE D. *A number is divisible by* 9, *if the sum of its digits is divisible by* 9.

Take the numbers 45678.

$$\begin{aligned} 45678 &= 40000 + 5000 + 600 + 70 + 8 \\ &= 4\,(9999 + 1) + 5\,(999 + 1) + 6\,(99 + 1) + 7\,(9 + 1) + 8 \\ &= (4 \times 9999 + 5 \times 999 + 6 \times 99 + 7 \times 9) + (4 + 5 + 6 + 7 + 8). \end{aligned}$$

Now the first bracket clearly is divisible by 9; therefore the whole number is divisible by 9, if the last bracket is

divisible by 9, that is, if the sum of the digits is divisible by 9. This agrees with the rule.

A similar proof would apply to the number 3.

A number is divisible by 11, *if the difference of the sums of the digits in the odd and even places is divisible by* 11.

Take the number 865432.

$$\begin{aligned}865432 &= 800000 + 60000 + 5000 + 400 + 30 + 2\\ &= 2 + 3\,(11 - 1) + 4\,(99+1) + 5\,(1001 - 1) + 6\,(9999 + 1)\\ &\qquad + 8\,(100001 - 1)\\ &= (2 - 3 + 4 - 5 + 6 - 8) + (3 \times 11 + 4 \times 99 + 5 \times 1001\\ &\qquad + 6 \times 9999 + 8 \times 100001).\end{aligned}$$

Now the second bracket is divisible by 11 (because 11, 99, 1001, 9999, 100001, are each divisible by 11); therefore the whole number is divisible by 11, if the first bracket is divisible by 11, that is, if the difference of the sums of the digits in the odd and even places is divisible by 11. This agrees with the rule.

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CONTENTS.

	Page
CLASSICAL	3
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